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A CROSS-SECTIONAL ANALYSIS OF PRODUCTION IN THE ALBERTA SAWMILL INDUSTRY

BY

TIMOTHY BRUCE WILLIAMSON

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
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OF MASTER OF SCIENCE
IN
FOREST ECONOMICS

DEPARTMENT OF RURAL ECONOMY

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SPRING, 1983

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled A CROSS-SECTIONAL ANALYSIS OF PRODUCTION IN THE ALBERTA SAWMILL INDUSTRY submitted by TIMOTHY B. WILLIAMSON in partial fulfilment of the requirements for the degree of Master of Science.

William E. Phillips

Supervisor

James Beck Jr.

Michael Carroll

Michael Percy

Date April 12, 1983

ABSTRACT

With the realization of a tremendous forest resource development potential and the consequent expectation of an expanded role for forestry within a more diversified Alberta economy, there is a need for more information describing structural input-output relationships within the forest industry. In response to such a need this thesis has three broad objectives:

1. to evaluate the use of various methodologies available for quantifying the structural relationships;
2. to provide empirical measure to the structural relationships and to indicate the relevance of the measures in terms of the pursuit of possibly desirable socio-economic objectives; and
3. to set the stage for further analysis with more flexible (but more complicated) functional forms.

Three specific mathematical models are evaluated: the Solow model, the CES trend variable specification, and the CES dummy variable specification. Although the Solow model shows that technological variegation contributes to cross sectional productivity differences, the results provided by the CES functional form show that the inherent inflexibilities of the Solow model result in an underestimated and improperly categorized measure of technological variegation. In comparing the dummy variable specification and the trend variable specification of the CES form, the former provides a better fit of the data and is more flexible in terms of allowing for the qualitative influence of size. The trend variable specification is more conducive to the measurement of the bias of embodied technological variegation. The choice of a particular functional form depends on the intent of the

analyst.

The empirical results provided in this thesis indicate that the Alberta sawmill industry's production technology is characterized by:

1. returns to scale of .88, .77, .71 and .65 for large, medium, small and smaller sawmills respectively;
2. technological differences, which are labour saving in larger mills, show a bias of +.072, between each of the 17 size categories; and
3. an elasticity of substitution between capital and labour which is .946.

These parameters have important implications in terms of the effective implementation of policy by instruments such as quota allocation, subsidization, and public participation in research and development.

A logical extension of the research endeavors in this thesis is the expansion of the analysis to include more than two inputs. Such an analysis however requires a more flexible functional form than those employed in this thesis. The necessary flexibility is provided by the transcendental logarithmic production function. Two additional inputs; stumpage and energy, could provide very useful information if incorporated into the production function specification.

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I. INTRODUCTION

A. The Study

With the expectation of an enhanced role for the forest industry in generating wealth, creating jobs and generally achieving regional development objectives, there is a need for information regarding the structural and technological characteristics of the industry. The general purpose of this thesis is to provide such information for the Alberta sawmill industry and to evaluate various methods available.

Specifically, the thesis quantifies and analyzes a number of empirical measures which describe the nature of the relationships between factors of production and lumber output. Because they directly impact the effectiveness of government policy, the measures are significant. Since production functions are statements quantifying the functional relationships between inputs and outputs, they are the principle mode of analysis used in the study.

B. Background

The forest industry is and traditionally has been a significant but quiet contributor to the Canadian economy. In terms of public awareness, other higher profile resource sectors such as agriculture and petroleum share the stage. In terms of social priority, these high profile sectors generally take precedence. However, since the conception of the nation, the Canadian economy has been and remains heavily dependent on the material wealth generated by the harvest of Canadian forests.

Evidence of Canada's early dependence is provided by Nelles (1974, p. 181) who states:

> "Between 1867 and 1899, bonuses, dues and ground rent from the lumber industry produced in excess of $29 million, or approximately 20 per cent of the total provincial (Ontario) revenue. Only the federal subsidy brought in a larger sum. In large measure the flourishing state of Ontario's public finances after confederation can be traced to this extraordinary income from forest regulation."

Ontario's natural resources, in particular the forest, abrogated any need for direct taxation of the province's citizens. Government services were, to the main extent, funded from revenues generated by the publicly owned resources (Nelles, 1974).

Evidence of Canada's present reliance on the forest industry is reflected in the following statistics (Economic Council of Canada, 1982).

1. direct employment of 310,000 workers;
2. total value of sales in 1980 of $22 billion; and
3. total value of exports of nearly $13 billion which comprised 17% of total Canadian exports in that year. (The export value of fish, grain and other farm products in 1980 was around $8 billion).

The relative importance of the resource in Alberta, as measured by jobs, value of sales, and export value, is not as pronounced as at the national level. However, it is significant. In 1978, 6,417 direct man years of employment were generated by the industry. Twenty-six Alberta communities were dependent on the economic activity generated by the industry. Total value of sales was $430 million of which $338 million

worth of product was exported out of the province (Ondro and Williamson, 1982).

Historically, the forest resource was important as a provider of the basic materials necessary for the developing provincial economy. In the 1880's, when C.P.R.'s trans-continental railway was constructed through the province, ties were manufactured from wood harvested from Alberta's abundant forests (Teskey and Smyth, 1975). In the 1900's, when settlement activity was increasing, Alberta forests provided the materials for shelter construction and fuel for heating the buildings.

Strictly by sheer dominance of the landscape, Alberta's forests attract attention. The total land area of Alberta is 644,547 km². Sixty-nine percent of this area is forest land. The total volume of timber on forest lands is 1.6 billion m³, of which softwoods comprise 52% and hardwoods 48% (Ondro and Williamson, 1982).

The future of industrial forestry in Alberta looks optimistic. Alberta has the second lowest proportion of utilized softwood annual allowable cut of any province in Canada (Table 1.1). There is also considerable potential for expanded use of Alberta's previously unutilized hardwood resource. Recent developments in processing and marketing of products from aspen have resulted in the construction of a large "oriented strand board" mill at Edson. This mill employs 200 people (Thomas, 1982). Also, research has shown that the use of aspen in a recently developed chemical thermal mechanical pulping process is an economically viable process. A recent report by Woodbridge, Reed and Associates (1981) suggests that Alberta has the resource potential to support an industry employing 6,000 people.

TABLE 1.1

Proportion of Softwood Annual Allowable Cut Unutilized, By Province

Province	Allowable Annual Cut (1979)	Harvest (Avg. 1978/79)	% of A.A.C. Utilized
	M m³	M m³	%
Newfoundland	2,940	2,594	88
Prince Edward Island	150*	150	-
Nova Scotia	3,273	3,804	116
New Brunswick	6,790	7,577	112
Quebec	36,000	28,352	79
Ontario	26,720	17,529	66
Manitoba	6,076	1,715	28
Saskatchewan	3,500	3,230	92
Alberta	14,639	7,170	49
British Columbia	73,483	75,199	102
Yukon	125*	125	-
N.W.T.	54*	54	-

*No A.A.C. figures are available; therefore the value is equivalent to Avg. harvest.

SOURCE: Department of Environment, September 1982, Discussion Paper: A Forest Sector Strategy for Canada.

The historical, current, and potential importance of the forest industry warrant its close scrutinization. Unfortunately, very little economic information is available regarding the nature of the forest industry. Economic analysis of the industry is largely lacking. The information provided by economic analysis is essential for not only monitoring the efficiency and competitiveness of the industry, but also for rational policy decision-making in the face of a range of options.

The scope of this thesis is limited to consideration of one component of the forest industry (sawmill industry) in one region of the country (Alberta). Specifically, the nature of the consideration is to describe the technological relationships between inputs and outputs and to examine likely policy ramifications of these relationships. The relationships between inputs and outputs is described in terms of a number of concepts which govern production. These concepts are definable as:

1. technical efficiency and technological variegation;
2. capital intensity of production process;
3. elasticities of substitution; and
4. returns to scale.

The vehicle for determining the nature and/or extent of these concepts is the production function. Production functions are mathematical formulations which provide empirical values describing the relationships between variables. The implicit functional form for the Alberta sawmill industry is:

Eq. 1.1 $Y = f(x_1, x_2, \ldots xn) \qquad i = 1 \ldots n$

where: Y = Output by the sawmill industry

Xi = some particular input or factor of production

n = number of unique inputs required for producing lumber.

The remaining portion of this chapter has four sections. The first section elaborates on the general problem. The second section specifies the objectives of the thesis. The third section provides a general overview of the methodology. The final section describes how the thesis is organized.

C. Problem Statement

The objective of "optimal resource utilization" can be a primary goal of resource managers. In one sense the firm is a resource manager in that it endeavors to organize its available resources (factors of production) in the most economically efficient manner. The motivating force is often profit maximization. The result is the most efficient use of resources which is socially desirable.[1] However another definition of a resource manager is also used herein as an entity that manages and allocates a public good (the forest).[2] An objective of the public good resource manager may be to maximize the net benefits which the resource provides to society.

In a freely working, perfectly competitive, open economy, optimal resource allocation is ensured by the market system.[3] In Alberta however, the market cannot be depended on to allocate forest resources to society's maximum net benefit. The public good nature of the resource necessitates a need for government intervention in managing and

---

[1] This statement assumes there is no need to redistribute income.

[2] Because of the large proportion of publicly owned land in Alberta the province's forests are public goods. In other areas of Canada and the U.S. forested lands may be private goods.

[3] This is a basic economic axiom which states that, under a number of simplifying assumptions, the market allocates resources so that they are put to their best use.

allocating forest resources. For effective management, governments must understand how policies affect the extent and distribution of goods and services provided by Alberta's forest resource. Therefore, government must be aware of the affect of policy on both the providers and on the users of the goods and services. The sawmill industry is both a provider, and a user of the inherent resource utilities. The industry is a user in that it depends on the in situ resource as a primary, raw material input. The industry is a provider in that it supplies goods to consumers who demand the utilities provided by forest products.

Before developing the problem statement further, an elaboration of the role of economic analysis for natural resources policy formulation and management is offered. The role of economics is stated by Howe (1979, p. 15) who suggests:

> "We prefer to think of economics as analytical in the sense that it is supposed to produce information on the implications of alternative policies, projects, or practices. Today, public decision-making is explicitly multiobjective. Officially and unofficially, the nation seeks various objectives through choices about resource use: economic growth, environmental enhancement, equity in the distribution of economic well-being and power, a satisfactory degree of independence of foreign sources, and so on. A democratic, informed decision-making process must be informed about possible alternative resource-use policies and their implications."

Governments at both the federal and provincial levels regularly make decisions which have profound effects on the industrial users of the forest resource. Modes of government involvement include: quota

allocation, collection of resource rents, capital subsidization, labour subsidization, product subsidization, loan guarantees, direct equity ownership, construction of infrastructure, tariffs, import and export quotas, research and development, regulations regarding use and renewal of the forest resource. The levels and types of methods employed are policy decisions, the effectiveness of which are subject to having a complete understanding of the affects of alternative policy decisions. In order to understand the effects of government policy, decision makers must have a clear understanding of the technological characteristics of the user industry.

The problem is a general lack of information regarding production relationships, with respect to the Alberta sawmill industry. A partial approach that deals with this problem involves answering three specific questions:

1. What are the functional relationships between factors of production and product in the Alberta sawmill industry?
2. What is the most appropriate technique for empirically quantifying the relationships?
3. What is implied by the results with respect to pre-defined social objectives and resource allocation policy alternatives?

D. Purpose and Objectives

The specific objectives of this thesis are:

1. To determine whether returns to scale are increasing, decreasing or constant for the sawmill industry;
2. To separate the effects of technological variegation on productivity

from increasing capital intensity of production processes;

3. To establish whether or not cross-sectional technological variegation in sawmilling is labour augmenting, capital augmenting or product augmenting;
4. To determine the influence of qualitative features such as scale of production and vintage of equipment on the parameters of production;
5. To determine the ease of substitution of capital for labour in the production process by way of an elasticity of substitution;
6. To compare alternative functional forms and, in light of the purpose of the analysis, data availability and extent of potential biases, to make recommendations as to the most appropriate functional form for future analysis;
7. To demonstrate applications of production function analysis in terms of policy instrument decisions regarding;
    a) alternative quota allocation schedules,
    b) nature, extent and delivery of alternative subsidization programs, and
    c) alternative decisions regarding levels of research and development expenditure;
8. To suggest a method for a more rigorous analysis of production which allows for direct inclusion of the forest resource as a factor of production in the manufacturing process.

E. Approach

The approach adopted in this thesis is one in which two separate economic models are developed based on two unique, explicit, neoclassical production functions. The first model is based on the Cobb-Douglas functional form (Cobb and Douglas, 1928). The generalized form of the Cobb-Douglas function is represented by:

Eq. 1.2 $Y = A \cdot K^{(1-\alpha)} . L^{\alpha}$

where: Y = sawmill output

A = technical efficiency parameter

K = capital input

L = labour input

α = a production elasticity describing labour's share of total output.

The second model is based on the Constant Elasticity of Substitution (C.E.S.) production function (Arrow, et al, 1961). The explicit form of this function is:

Eq. 1.3 $Y = \gamma[\delta \cdot K^{-\rho} + (1 - \delta) \cdot L^{-\rho}]^{-\tau/\rho}$

where: Y = sawmill output

K = capital input

L = labour input

γ = technical efficiency parameter[4]

δ = capital intensity parameter[4]

ρ = substitution parameter[4]

τ = returns to scale parameter[4]

Details of the theoretical significance of the parameters are provided in Chapter III. Details regarding model construction and

---

[4] When referred to collectively, these parameters are henceforth termed parameters of production.

estimation procedures are provided in Chapter IV.

F. Sequence of Chapters

The following chapter provides a detailed descriptive analysis of the sawmill industry in Alberta. The information provided takes two dimensions. First, the industry is characterized by indicating the size and structure of the industry and the technology it employs. Second the economic contribution of the industry is indicated. The third chapter describes the basic theory of production. Emphasis is placed on appreciating the significance of the production concepts (i.e., returns to scale, capital intensity, technological variegation, and elasticity of substitution). The concepts are developed in general terms using forest industry examples. The fourth chapter describes the methodologies employed in the analysis. For each model, the model construction, estimation technique and statistical properties are described. The fifth chapter describes the source, collection techniques and statistical representativeness of the sample data. The sixth chapter presents the results of the models described in the fourth chapter. The results are also interpreted in light of some of the objectives of the thesis. The seventh chapter provides a general summary of the overall research. The implications of alternative resource allocation policies are provided and avenues for future research described.

II. THE SAWMILL INDUSTRY - A DESCRIPTIVE ANALYSIS

A. Introduction

The "forest industry" in Alberta is comprised of 23 component sectors. The industrial activities characterizing these sectors involve the primary, secondary and tertiary manufacturing of harvested roundwood. Table 2.1 lists these sectors along with their appropriate four-digit standard industrial classification (SIC) codes. To term the sectors collectively as being a single industry is a misnomer. An industry is a group of firms producing a homogeneous output by relatively similar methods of production. Thus each of the sectors listed in Table 2.1 is a unique industry.

Primary wood-using industries are those industries that convert roundwood into a utility bearing product. Gregory (1972, p. 89) states, "Primary manufacturing is concerned with the initial treatment of the harvested raw material." Primary wood-using industries in Alberta include the sawmill and planing, veneer and plywood, wood preservation, pulp and allied,[5] and miscellaneous[6] industries. This study focuses exclusively on the sawmill industry.

B. Industry Profile[7]

Gregory (1972, p. 101) describes the general nature of the American sawmill industry with:

"lumber takes nearly half of the total cut from United States

---

[5] Allied includes fibreboard, building paper, and roofing products.

[6] Miscellaneous industries include pallet, lath and building timber and log manufacturers.

[7] All the descriptive information provided in this chapter was obtained from the Northern Forest Research Centre's Survey of Primary Wood Using Industries 1978-79 (C.F.S., 1982).

TABLE 2.1
Component Sectors of the Alberta Forest Industry

Industry	S.I.C. Code
1. Sawmills and Planing Mills	2513
2. Veneer and Plywood Mills	2520
3. Sash, Door and Other Millword, N.E.S.	2541
4. Pre-Fabricated Buildings (Wood-frame)	2543
5. Manufacturers of Wooden Kitchen Cabinets	2544
6. Wooden Box Factories	2560
7. Coffin and Casket Industry	2580
8. Wood Preservation Industry	2591
9. Miscellaneous Wood Industries, N.E.S.	2599
10. Furniture Re-Upholstery and Repair	2611
11. Household Furniture Manufacturers, N.E.S.	2519
12. Office Furniture Manufacturers	2640
13. Miscellaneous Furniture and Fixture Manufacturers	2660
14. Pulp and Paper Mills	2710
15. Asphalt Roofing Manufacturers	2720
16. Folding Carton and Set-Up Box Manufacturers	2731
17. Corrugated Box Manufacturers	2732
18. Paper and Plastic Bag Manufacturers	2733
19. Miscellaneous Paper Converters	2740
20. Commercial Printing	2860
21. Plate Making, Typesetting, Etc. Ind.	2870
22. Publishing Only	2880
23. Publishing and Printing	2890

Source: Alberta Bureau of Statistics, 1981. Alberta Principal Manufacturing Statistics - 1978. ISSN 0382-1951, Alta. Treasury, Edmonton.

forests, is produced by thousands of independent, highly individualistic operators in mills ranging from units producing a hundred thousand board feet of badly made, undried, ungraded lumber to those manufacturing annually a hundred million board feet of carefully graded, sized, kiln dried, and packaged construction material."[8]

This description is entirely appropriate for describing the Alberta sawmill industry. There were 394 sawmills in Alberta in 1978 ranging in annual output from 5 thousand fbm to 90 million fbm. Table 2.2 shows the distribution of these mills along with output levels by pre-defined size groups. Although mills producing greater than 1 million fbm per year are in the minority, they account for 94% of the total production of the industry. Total industrial output in 1978 was over 750 million board feet.

Table 2.3 shows that the largest six mills produce over 50% of the total physical output by the industry. This level of concentration suggests the possibility that the firms are oligopolists and or oligopsonist and that they could therefore have some collective influence on either lumber values or input prices. However since the firms are competing on the large open North American lumber market where they effectively have no market power, suspicion of any oligopolistic influence is unfounded. Similarly, on the factor supply side, firms in the sawmill industry must compete with many other resource sectors (such as other primary wood-using industries, oil and gas, coal, and agriculture) for factors of production. This competition is particularly

---

[8] "This does not mean that all small mills produce poor lumber, or that all large mill lumber is correctly manufactured."

TABLE 2.2
Total Production and Number of Mills by Size Class

Size Class (Annual Output - M fbm)	No. of Mills	Total Lumber Production M fbm
greater than 5,000	26	649,870
1000 - 5000	29	61,608
100 - 1000	128	38,576
less than 100	211	8,668
Total	394	758,722

Source: Canadian Forestry Service (1982)

TABLE 2.3
Concentration of Production in the Sawmill Industry

Cumulative No. of Mills	Cumulative % of Mills	Cumulative % of Total Production
3	.8	32.9
6	1.5	50.7
9	2.3	60.5
16	4.1	75.4
28	7.1	87.6
55	14.0	94.0
183	46.4	98.9
394	100	100

Source: Canadian Forestry Service (1982)

true in the labour market where relatively unspecialized[9] labour requirements in sawmills diminish the threat of oligopsonistic control of wage rates.[10]

As Gregory suggests for the U.S. sawmill industry, product quality in Alberta's smaller sawmill classes is generally of an inferior nature. Table 2.4 shows that the proportion of higher quality planed lumber increases in each successively larger size class. The proportion of product exported increases in larger size classes. Small sawmills generally satisfy local market demands with a lower quality product than is required for competing in the international market to which production in the larger size classes is geared.

Further insight into the cross-sectional character of the sawmill industry is provided by investigating ownership types by size class. From Table 2.5 the predominance of limited liability corporations in sawmills producing greater than 1 million fbm is evident. In the smaller sawmills, single proprietorship ownership prevails. Average age of current ownership declines in smaller sawmill size classes indicating a pattern of less stable proprietorship.

The emerging scenario is one portraying a concentrated yet highly diverse industry with large national or multi-national firms producing a high quality product for sale on international markets on the one hand, and small, single proprietorship, bush sawmill operations, producing lower quality, ungraded lumber for sale on local markets on the other

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[9] This characteristic is becoming debatable with the innovation of increasingly complex conversion systems requiring highly skilled and technically trained personnel for their operation and maintenance.

[10] Wage rates vary considerably in the Alberta sawmill industry and are in general lower in the small owner operated sawmills and higher in the larger mills.

TABLE 2.4
Percent of Total Product Planed and Percent Exported Out of Province by Size Class

Size Class (Annual Output - M fbm)	% of Total Product Planed	% of Total Product Exported
greater than 5,000	94%	80%
1000 - 5000	63%	8%
100 - 1000	30%	2%
less than 100	13%	0%

Source: Canadian Forestry Service (1982)

TABLE 2.5
Ownership Characteristics of Alberta Sawmills

		Type of Ownership				
					Corporation	
Size Class	Average Years Ownership	Single Proprietership	Partnership	Provincial	National	Foreign
5000 +	15.2	-	-	16	6	4
1-5000	12.1	4	-	25	-	-
100-1000	11.7	85	14	29	-	-
100 -	10.0	188	14	9	-	-

Source: Canadian Forestry Service (1982)

hand. This diversity in product and ownership is also manifest in methods of producing lumber in the sawmill industry. The types and variability of production processes is addressed in the following section.

C. Current Technology[11] and Methods of Producing Lumber

The subject of technology and production methods is inextricably linked to an industry's infrastructure. This section provides a detailed investigation of the form and vintage of the sawmill industry's capital stock. First, consideration is given to the form of production technology employed by Alberta sawmills.

As previously mentioned processing methods vary widely across the spectrum of Alberta sawmills.[12] For the larger mills, "state of the art" technology is generally necessary to remain competitive. Two separate sources describe the nature of this technology. The Department of Industry, Trade and Commerce (1979, p. 30) states:

> "During the 1960's, in an effort to reduce unit logging costs and to comply with the application of more stringent utilization standards on crown land, the industry in the B.C. Interior and Central Canada converted extensively to more intensive harvesting methods such as tree length logging which resulted in large volumes of small size material at the sawmill site. At the same time, new sawmilling technology including profile chipping machines, high strain band mills and thin kerf circular saw equipment was being developed with the capability of processing

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[11] For complete explanations of sawmill technology see Mullins and McKnight, (1981) and Williston, (1976).

[12] This variability is termed technological variegation in later parts of the thesis.

> small diameter logs at high linear feed rates. Attention was then focused on the development of total systems designed for small log processing. These systems have greatly lowered the minimum size of timber that can be economically processed into lumber and has allowed industrial expansion to take place in the more northern and marginal timber areas."

John Gray (1981, p. 10) states:

> "Small-log sawmills are designed to handle logs on a production-flow "one-pass" basis, rather than individually and on a "multiple-pass" with carriage headrig mills. They utilize multiple band-saw headrigs (twin-band or quad-band), multiple-circular-saw (scrag saw) headrigs, or chipper-canter headrigs of various types, to break down the logs into lumber, or cants. The pattern is set to conform to the log size and to produce maximum lumber recovery on a single pass. The lumber is further processed by other machinery: multiple-saw gang edgers, and resaws. The head saw operates at high speed. Mill capacity is dependent not only on log diameters but more importantly on the linear footage throughput."

Table 2.6 shows the distribution of lumber processing equipment in sawmills producing greater than 5 million fbm per year in 1979. Clearly, the majority of the mills in this class rely on the rapid linear feed sawmill technology. Only 8 of the 26 mills exclusively use circular headsaws for primary log breakdown.

Observation[13]

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[13] The following reference provides a general listing of sawmill equipment in each of Alberta's sawmills - Ondro, W. J., B. W. Karaim, R. A. Bohning, and G. R. Stevenson. 1980. A Directory of Primary Wood-Using Industries in Alberta, 1979. Environment Canada., Can. For. Serv.,

TABLE 2.6
Distribution of Lumber Processing Equipment in Alberta Sawmills Producing Greater Than 5 MM fbm, in 1979

Type of Equipment

Mill No.	Debarkers	Cut-Off Saws	Circular Headsaws	Scrag, Band, Gang and Quad saws	Chip-N-Saw Chipping canter	Reman or Resaws	Edger and Trim saws	Chippers	Chipping Edger	Planer	Dry Kiln
1	1		1				2	1		1	
2		1	1	2		1			1	1	1
3	2	1	1	1	1	1	1	2		1	2
4	2	2		2		3	3		1	1	2
5			1	1		1	2			1	
6			2				3			2	
7		1	1	1		1	3			1	
8		1	1				2				
9	2	1		1	2		1			2	3
10	1	1	1	1		1	2			1	2
11				3			1			1	
12		1	1	2		1	3				
13	1	1	1	2		1	1	1		1	1
14		1	1			1	1			1	
15		1	1			1	2			1	
16	4	4			2		2		2	1	3
17		2	1	1		1	2			1	
18	3	1		2		2	2	1		1	
19				1			3			1	1
20	1	2		1		1	3	1			
21	3	4			3	1	8			2	5
22			1			1	2				
23		1	1			1	1			1	
24		1	1			1	2			1	
25	2			1	1	1	1		1	1	
26	1	2		1			1	2		1	1

Source: Ondro et al., 1980

of sawmill equipment among smaller sawmills shows a decreasingly sophisticated sawmill technology with increasingly smaller mills. Sawmills in the 1-5 million fbm per year class primarily use circular saw headrigs for log breakdown. However, some mills are more progressive because they utilize the more modern methods desribed previously. Also, some of the mills with circular headsaws are the high-speed, thin-kerf variety.

Sawmill technology among mills producing less than 100 thousand fbm per year is characterized by small circular headrig mills which in many cases have been rebuilt from delapidated mills or constructed entirely from spare parts. The power source for many of these mills is supplied by power take offs from farm tractors.

Another factor characterizing the form of sawmill machinery is the portability of equipment. Table 2.7 shows that a higher proportion of sawmills are portable in each successively smaller sawmill class. For certain levels of mill operations, portable sawmills are the most cost effective method of conversion. The reduced unit logging costs associated with transferring the mobile mill to the raw material source offsets other cost disadvantages for certain mill operations. However in larger sized sawmills the economies associated with stationary sawmills preclude the use of portable mills.

Vintage of production equipment is an additional characteristic of an industry's production technology which merits investigation. The existence of an older capital stock has obvious implications as to the progressiveness of the industry's production methods. Table 2.8 shows the average age of plant machinery by sawmill size class. The trend is

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[13] (cont'd)North. For. Res. Cent. Inf. Rep. NOR-X-220.

TABLE 2.7
Portability of Sawmills in Alberta - 1978

Size Class (Annual Output - M fbm)	No. of Stationary Mills	%	No. of Portable Mills	%
Greater than 5,000	26	100	0	0
1000 - 5000	25	87	4	13
100 - 1000	95	74	33	26
Less than 100	39	39	61	61

Source: Canadian Forestry Service (1982)

TABLE 2.8
Average Age of Plant Machinery and Equipment in Alberta Sawmills - 1978

Size Class (Annual Output - M fbm)	Average Age
Greater than 5000	8.7
1000 - 5000	13.0
100 - 1000	19.3
Less than 100	21.9

Source: Canadian Forestry Service (1982)

toward an older vintage of capital in each progressively smaller sawmill class. Thus the proportion of mills using recently developed technologies is much lower in small mills than in large mills and small mills are less apt to take advantage of productivity improving technological changes.

This section has shown the variability in the form and vintage of capital used by the sawmill industry. A few general conclusions have been made as to the nature of the industry in terms of its structure, and product and process diversity. The general conclusions lead to a more complex series of empirical questions which, when answered, will enhance the basic understanding of the industry's technological relationships.

D. Economic Importance of the Industry

Figure 2.1 shows that the Alberta sawmill industry is one small part of a large, resource extraction based economy. The $89.7 million total output produced by the industry accounted for only .3% of total Gross Provincial Product (G.P.P.) in 1978. This fact, however, should not belittle the importance of the industry because very few single, four-digit[14] level industries would show a significantly greater impact. Therefore, this section does not dwell on the relative importance of the industry but rather on its absolute contribution.

A number of criteria, each of which is important but none of which is all-encompassing, can be used to describe an industry's economic impact. Quantitative variables relaying the economic contribution of the sawmill industry include total industry sales, value-added, number of

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[14] This classification shows the level of the industries disaggregation according to Statistics Canada's S.I.C. code.

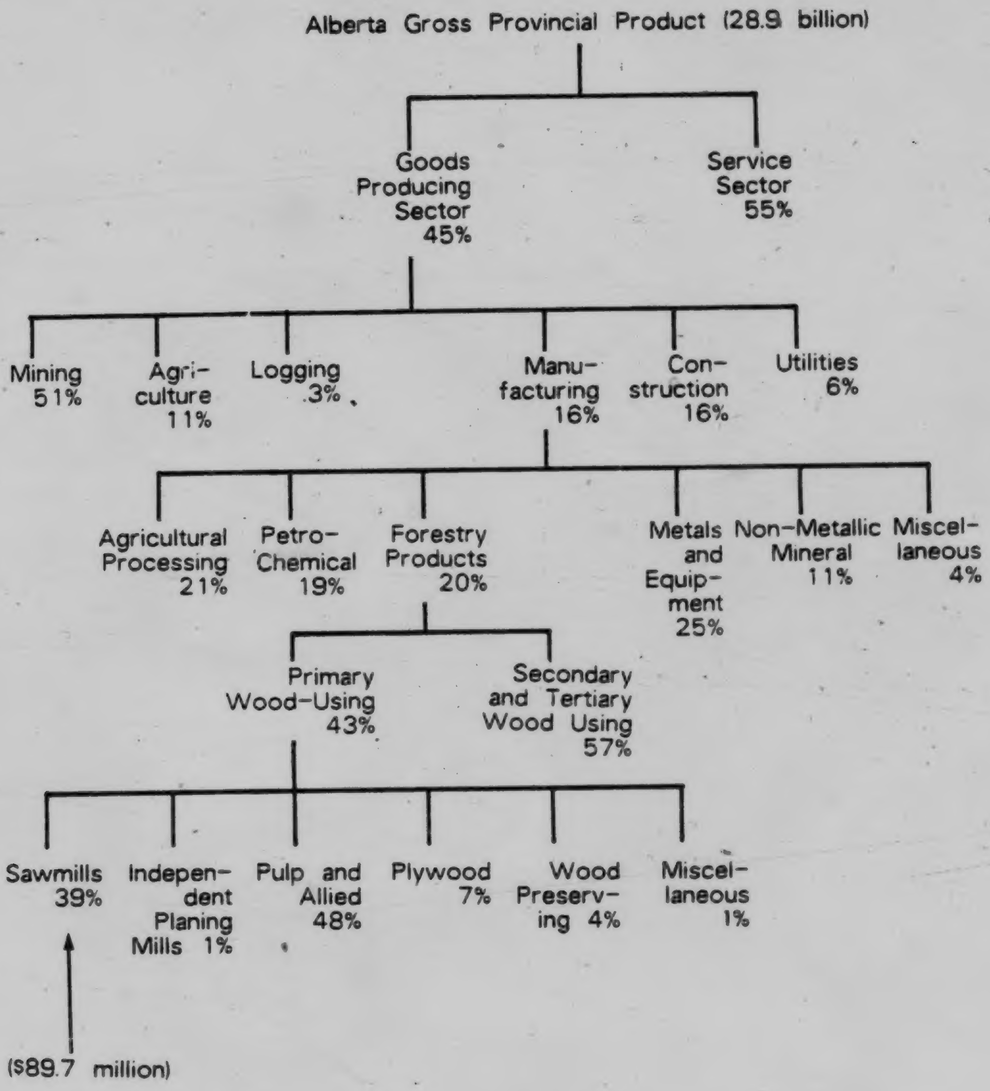

FIGURE 2.1. Structure of the Alberta Economy- 1978

Source: Alberta Bureau of Statistics (1980, 1981)

jobs created, wages and salaries paid, and value of extra-provincial exports.

Total sawmill industry sales in 1978 were $159,441,569. Seventy-nine percent of this value was provided from the sale of goods to out-of-province markets, thus indicating the "base sector" nature of the industry. Total industry value-added in 1978 was $89,723,091. This value represents the industry's contribution to G.P.P. and shows the total payments to primary factors of production (capital, labour). If these factors are locally (within the province) supplied then the value represents the contribution to provincial wealth.

The sawmill industry also created 3,586[15] direct jobs[16] in 1978 (4.8% of total manufacturing sector employment). Total wages and salaries paid were $45,303,696 (4.0% of total manufacturing sector wages and salaries paid). If an estimate of income paid to associated contract[17] workers is included, wages and salaries were equal to $56,963,286. Industry payments to capital (depreciation and profits) can be estimated by netting out payments to labour from total value-added. Thus total payments to capital are estimated as $44,419,395.

This chapter provides some perspective as to the economic impact and cross-sectional character of the Alberta sawmill industry. The descriptive analysis is essentially a "snap shot" of the contemporary industry. Indications of the inherent diversity are reflected by a cross-sectional variability in product quality, processing methods, ownership patterns and vintage of equipment. These general observations inspire a spirit of enquiry with regard to the empirical estimation of

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[15] Value includes contract loggers.

[16] These are full employment equivalents defined as a person working 8 hours/day, 5 days/week, 52 weeks/year or 2080 hours per year.

[17] Those employed by contract logging operations.

production relationships in the sawmill industry. Before this estimation can proceed, however, the theoretical basis of the relationships must be understood. Thus the following chapter provides a short overview of production theory particularly as that theory relates to the significance and estimation of the parameters of production.

III. THE THEORY OF PRODUCTION

A. Introduction

Stated simply, production is the reconstitution of inputs into outputs. The economic theory of production is essentially the study of the characteristics of these conversion processes, not from an engineering standpoint but in a more abstract sense. This stance allows the economist to simplify the complex, multi-faceted and highly diverse engineering technologies of the nation's industrial complex into a universal framework from which important and relevant conclusions regarding resource use and allocation can be ascertained.

Production theory addresses the behavior of the firm in attempting to explain how and why the firm allocates resources to achieve certain objectives. The theory is predicated on explaining the technological relationships between inputs and outputs. These relationships can be represented by the firm's production function. The purpose of this chapter is to explain the basic axioms endowed within the theory and to describe, in principle, how these axioms can be represented empirically.

B. Production With One Variable Input

A logical starting point for illustrating the underlying postulates of production theory is to consider the production map of a firm holding all inputs constant except one. The responses of output to variation of the single input are shown in Figure 3.1. Two important concepts are illustrated in Figure 3.1 including (1) the law of diminishing marginal productivity and (2) the three stages of production of the profit maximizing firm.

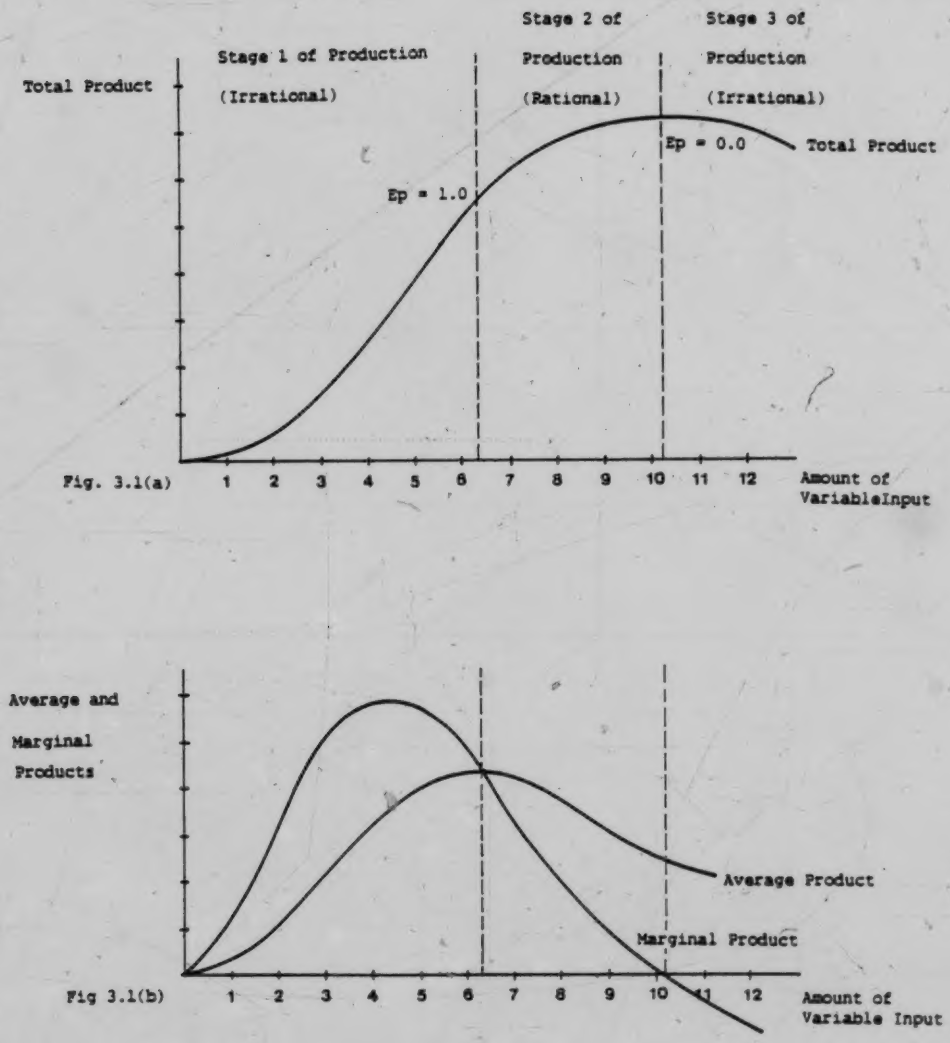

Fig. 3.1(a)

Fig 3.1(b)

FIGURE 3.1. Total, Average and Marginal Product Curves of a Firm Varying a Single Input

Figure 3.1(a) shows the increase in total output which is the result of increases in some factor of production (assuming all other factors of production are held constant). The curve is labelled the total product (TP) curve and it represents the returns to a single variable input. A feature of the curve is that total output increases with each successive increment of input. This trend continues up to a point where total output begins to decline with additional input. The point where TP reaches a maxima is defined as the "point of diminishing productivity"[18] (Stager, 1973) and is identified on 3.1(a) where the elasticity of production (Ep) equals zero.[19] Clearly, the employment of resources past this point is irrational since each additional unit of input provides negative product.

A derivation of the total product curve is average product.[20] Average product is derived from the TP curve by measuring the slope of a straight line projected from the origin to successive points on the TP curve. The resulting average product curve is shown in Figure 3.1(b). This graph demonstrates that as more units of the variable input are added to the fixed inputs the AP curve increases to a maximum and thereafter declines. The point of maxima is defined as the "point of diminishing average productivity" (Stager, 1973) and is identified on Figure 3.1(a) where the elasticity of production of the variable input is equal to one. At each level of input use less than the point where diminishing average productivity occurs, the elasticity of production of the variable input is greater than one. This is logical since if Ep is increasing with each successive addition of input, then the relative

---

[18] Note: this is not diminishing marginal productivity.
[19] Elasticity of production = Ep = (%Δ output) / (%Δ input).
[20] Average product = AP = Output/Input.

contribution of the input to output must be greater than the relative change in the input. Thus, %Δ output / %Δ input is greater than one. Clearly it is irrational for a firm to employ any level of input where Ep > 1, since a given proportionate change in the input provides a greater proportionate change in output. The region of the Tp curve where Ep > 1 is stage one of production. The miniumum level of variable input which the rational profit maximizing firm will employ occurs at the point where Ep = 1.0. Therefore stage one is irrational.

The derivative of the total product curve at any point is the inputs marginal product.[21] As previously suggested, the incremental addition of units of input have a positive effect on output at all levels of input use between the origin and the point where Ep = 0.0. Although the change in output associated with incremental changes in input is absolute, the magnitude of each production increase is not constant. In other words the rate of change of TP is not stable. The terminology used for representing the rate of change of the total product curve is marginal product. The marginal product of a variable input is equal to the increase in output associated with an increase in one unit of the variable input. Thus, from the first derivative (or slope) of the TP curve, at each level of input use, the marginal product curve is derived. The MP curve is plotted on Figure 3.1(b).

The last sentence of the first paragraph of this section suggested that two important concepts were illustrated in Figures 3.1(a) and (b). With regard to the first concept, the law of diminishing marginal productivity[22] is defined as:

If increasing amounts of a variable factor are applied to a

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[21] Marginal product = (Δ Output) / (Δ Input).

[22] Sometimes also referred to as the law of variable factor proportions.

> fixed quantity of the other factors, the amount added to the total product by each additional unit of the variable factor will eventually decrease; after this point has been reached, each additional unit of the variable factor will add less to the total product than did the previous unit. The level of output at which the amount added to total product by each successive unit of the variable factor starts to diminish, is called the point of diminishing marginal returns."
>
> (Lipsey, 1966, p. 181).

This law clearly expresses itself in Figure 3.1(b). The point of diminishing marginal productivity occurs where the marginal product curve is at a maximum.

With regard to the second concept three separate stages of production have been shown to occur. The first stage is irrational however since the firm will always want to add more inputs. Similarly the third stage is also an irrational use of inputs since more input decreases output. The only rational range of input use occurs between the points where the elasticity of production of the variable input is between zero and one. This area is identified as Stage 2 of production.

C. Production with Two Variable Inputs

The conceptual model developed in the previous section is an obvious simplification of real firm behavior. It did, however, serve to introduce some fundamental postulates. The conceptual model developed in this section enriches those postulates by both allowing for the inter-factor substitution of two variable inputs and by introducing certain economic magnitudes required for determining optimum input use.

The concepts developed in this section show the necessary and sufficient conditions for deriving the technically and economically efficient mix of inputs.

Figure 3.1 illustrated the basic input-output relationship of a firm wishing to vary a single factor of production. This approach, however, does not explain the behaviour of a firm varying two factor inputs. In this regard what is desirable is to illustrate the production relationships between inputs (factor-factor rather than factor-output). Central to this theme is the concept of substitutability of factors of production. The purpose of observing factor-factor relationships is to identify technically efficient combinations of resource inputs. A technically efficient input mix is a necessary condition for determining the economically optimal mix of inputs.

Before proceeding into the realm of factor-factor production relationships a visual illustration of the effect of multiple inputs on output is enlightening. The graphical representation of the effect on output is called a production surface. Such a production surface using capital, labour and lumber as examples of the inputs and outputs is shown in the three-dimensional graph of Figure 3.2.[23] The horizontal plane (ABCD) of Figure 3.2 represents an infinite number of combinations of capital and labour. The corresponding output associated with any particular combination of inputs is represented by the vertical distance from the horizontal plane to the face of the production surface. A set of points of equal vertical distance will, if enough points are plotted, form a contour line such as that indicatd by FG. This contour line

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[23] For perspective, the similarity between the curve AE on Figure 3.2 and the TP curve of Figure 3.1(a) is clearly evident. This is logical because they represent exactly the same relationship.

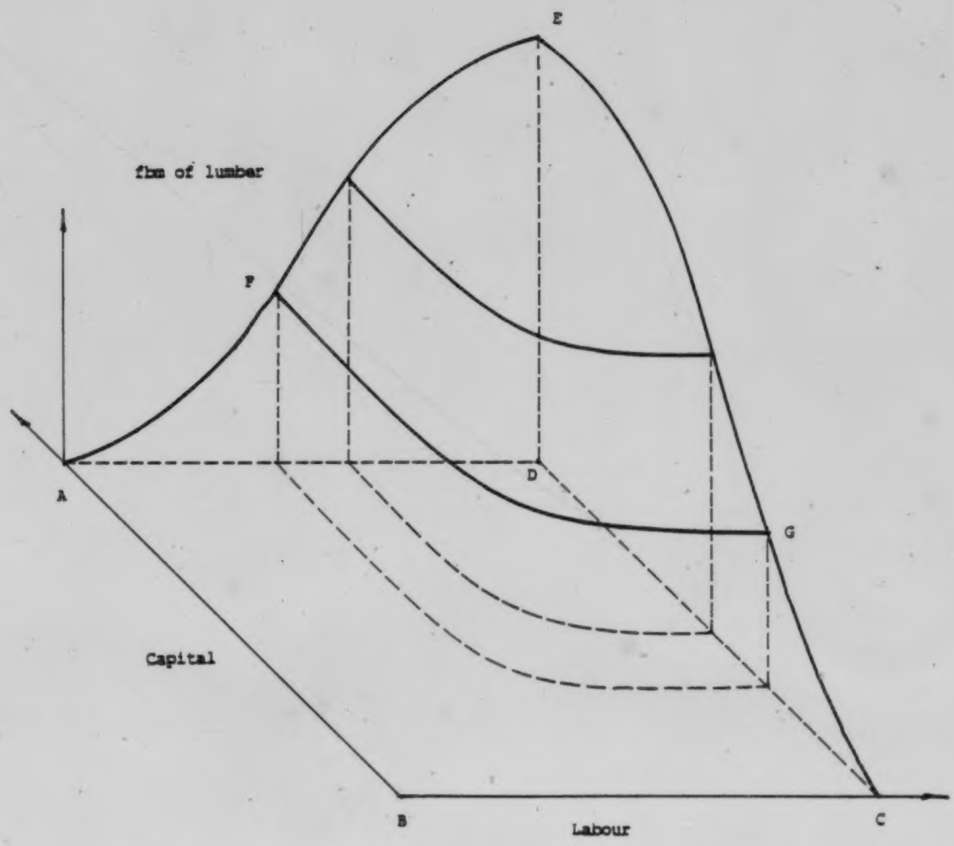

FIGURE 3.2. The Firm's Production Surface

represents a limitless number of technically efficient combinations of capital and labour each associated with a uniform level of output. Thus the line is called an isoquant (iso - the same, quant - quantity).

Isoquants provide an extremely useful forum for illustrating factor-factor production relationships and are thus the primary means of doing so. Isoquants represent the spectrum of technically efficient input mixes which can be used to produce a given level of output. To satisfy the necessary conditions of technical efficiency, the entrepreneur must chose from among the input combinations offered by the isoquant. Figure 3.3 illustrates the isoquant which corresponds to curve FG on Figure 3.2

Four features characterize the shape of isoquants. First isoquants are continuously differentiable. Second, they are non-intersecting. Third, rational segments of the isoquant slope downward to the right. Fourth, rational segments are convex to the origin.

The first feature provides for a smooth and continuous isoquant curve. This concept is however an abstraction in that the "lumpiness" that can occur in certain factors of production is ignored by assuming that inputs can be divided into any number of smaller units.

The second feature, implies that two levels of output could be produced with exactly the same mix of inputs at the intersection point. For firms utilizing resources efficiently this is clearly illogical since rational firms would never employ inputs that produce a lesser amount than is possible with some other production process that uses the same level of inputs.

The assumption that firms only choose efficient input mixes assures that for rational production only the input mixes on portions of

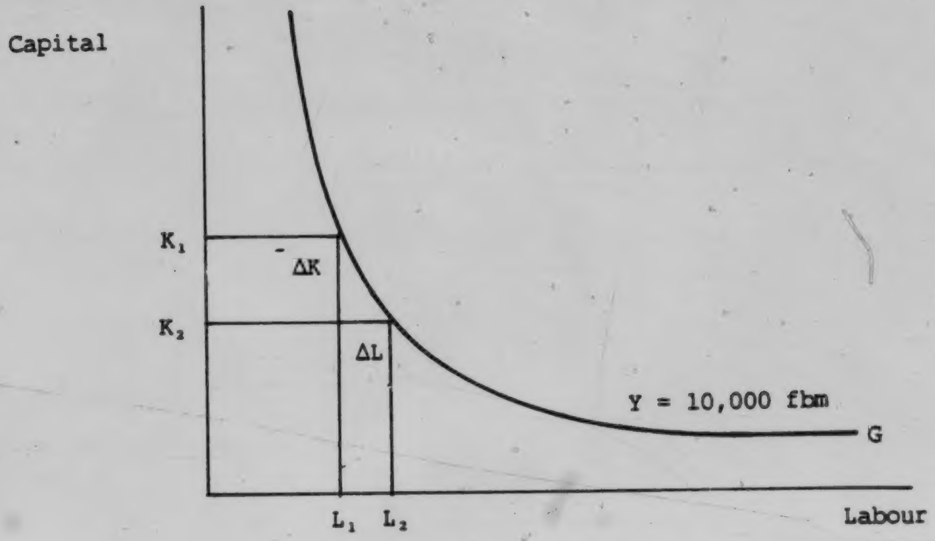

FIGURE 3.3. An Isoquant For a Sawmill Producing 10,000 fbm of Lumber a Year

the isoquant sloping downward and to the right are chosen. This statement stems from the options that the firm faces in substituting one input for another. The rate at which one input can be substituted for another is measured by the slope of the isoquant. This rate is termed the "marginal rate of technical substitution" (MRTS) of labour for capital. From Figure 3.3, the MRTS is derived measuring the increase in labour (ΔL) which is required to compensate a decrease in capital ($-\Delta K$) so that output remains constant. Thus Eq. 3.1 represents the MRTS (for small changes in L)

Eq. 3.1 $\quad MRTS = -\Delta K/\Delta L$

From Eq. 3.1, the negative magnitude of MRTS is evident. This indicates the downward sloping nature of the MRTS. Clearly if MRTS were positive then increases in both capital and labour are associated with constant levels of output. Thus all points on the isoquant where MRTS is positive are irrational segments and they will never be observed.

An interesting aspect is revealed by allowing output to change with variation of a single input (holding the other input constant). If capital were reduced from K_1 to K_2 on Fig. 3.3 and labour was fixed, the decrease in output would be equivalent to

Eq. 3.2 $\quad -\Delta K \cdot MPk$[24] $= -\Delta Q$

The gain in labour (L_1 to L_2) required to increase output to its previous level is:

Eq. 3.3 $\quad \Delta L \cdot MPl = \Delta Q$

Thus, since the gain in product is equivalent to the loss;

Eq. 3.4 $\quad -\Delta K \cdot MPk = \Delta L \cdot MPl$

or

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[24] MPk is the marginal product of capital and MPl is the marginal product of labour.

Eq. 3.5 $-\Delta K/\Delta L = -(MPl)/(MPk) = MRTS$

Thus the slope of the isoquant is equal to "the ratio of the marginal products of factor inputs" which "is equal to the marginal rate of technical substitution (MRTS)." The significance of this feature becomes apparent in Chapter 4.[25]

The final feature characterizing isoquants is that on the region where the MRTS is negative the curve is convex to the origin. Therefore, since the slope of the curve is measured by the MRTS, the portions of the curve described by a higher K/L ratio will also have a higher MRTS. The MRTS falls as, labour is substituted for capital and approaches zero on the flatter part of the curve. The explanation for this observation is based on certain input-output relationships described earlier in the chapter. Recall from Section 2 that the law of diminishing marginal product states that within the economic region of production, uniform increments of input result in progressively smaller output increments. The movement from left to right along the isoquant entails a continuous substitution of labour for capital. Thus as progressively more labour is used in the production process the MP of labour declines. Similarly as proportionately less capital is used, the MP of a unit of capital is increased. Therefore, since in moving along the isoquant the MPl is falling and the MPk is rising, the MRTS (which is defined by MPl/MPk) must be steadily declining. Thus, since MRTS measures the slope of the isoquant and since it steadily declines moving from left to right, the isoquant must be convex to the origin.

To reiterate, the isoquant indicates the position of all possible technically efficient combinations of inputs required to produce a constant level of output. A necessary condition for optimum input use is that the firm must choose an input mix located on the curve. Although technically eficient, this restriction does not guarantee that the economically optimum input mix will be provided. The profit maximizing firm is as much constrained by economic limitations as by technical limitations. Thompson (1973, p. 235) states;

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[25] See page 75.

> "firms are limited in their choice of production techniques by the prices of resource inputs and by the amount of funds available for purchasing these inputs. An isocost curve portrays the various alternative combinations of resource inputs which a firm can purchase given the prices of resource inputs and the stipulated amount of expenditures on resources."

Figure 3.4 illustrates an isocost line. The isocost line is defined by the expression

Eq. 3.6 $TC = Pl \cdot L + Pk \cdot K$

where TC = total cost

Pl = price per unit of labour

Pk = price per unit of capital

K = total capital input

L = total labour input

The intersection point on the vertical axis of Figure 3.4 is provided from

$$TC = Pl \cdot 0 + Pk \cdot K$$

Therefore $TC = Pk \cdot K$

Therefore $K = TC/Pk$

The intersection point on the horizontal axis is given by;

$$TC = Pl \cdot L + Pk \cdot 0$$

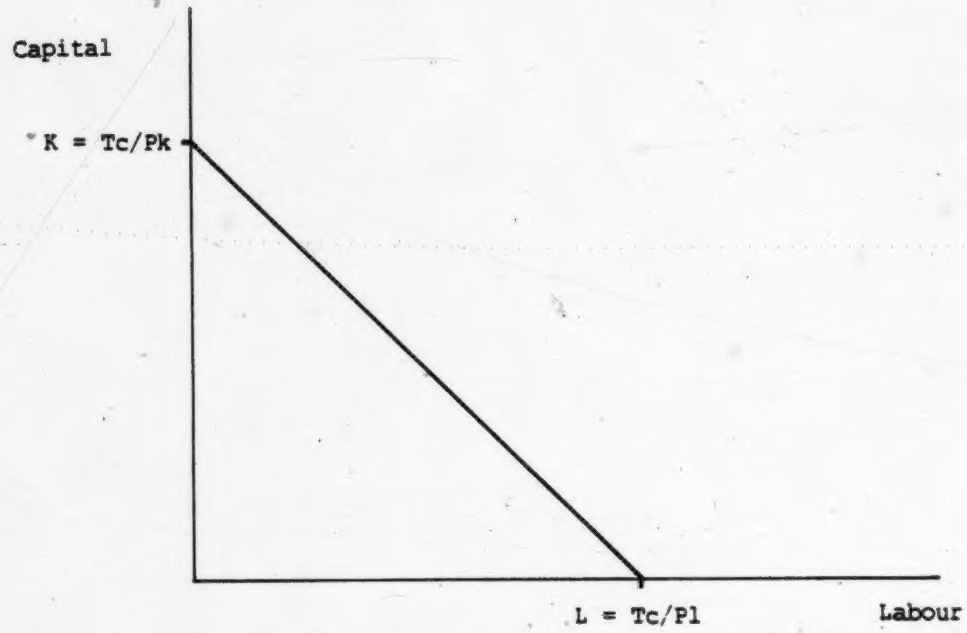

FIGURE 3.4. An Isocost Curve For A Sawmill Purchasing Capital and Labour

Therefore $TC = Pl \cdot L$

Therefore $L = TC/Pl$

The straight line[26] joining these two intersection points represents different combinations of the two resource inputs associated with a constant cost and is therefore termed an isocost line. The slope of the isocost line is a constant and is equal to

Eq. 3.7 $(-)(TC/Pk)/(TC/Pl) = (-)Pl/Pk$

The isocost line is a necessary prerequisite for selecting the economically optimal input mix from the spectrum of technically efficient input combinations identified on the isoquant. The optimal input mix is indicated at the point of tangency between the isocost and the isoquant. This point is shown at A on Figure 3.5. The corresponding optimal levels of capital and labour are K_1 and L_1. A logical question is: How does the firm conclude that the input mix indicated at A is optimal? Two separate viewpoints can be taken. If a particular firm endeavors to produce a predetermined level of output, the firm, being a profit maximizer, will attempt to minimize the costs of producing the output. From Figure 3.6, the minimum cost occurs at A. Although ten units of output can be produced with the input mix K_2/L_2 (which is technically efficient) the input mix K_1/L_1 still produces ten units but at a lower cost and is therefore preferred.

The alternative viewpoint is that a firm with a pre-determined fixed budget endeavours to maximize output. Thus the firm's managers will choose input mix K_1/L_1 on Figure 3.7 because this combination both exhausts the budget and maximizes output. The naive firm manager may

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[26] The isocost line remains straight only as long as the level of input use has no effect on input prices (See Thompson, 1973, footnote on p. 236).

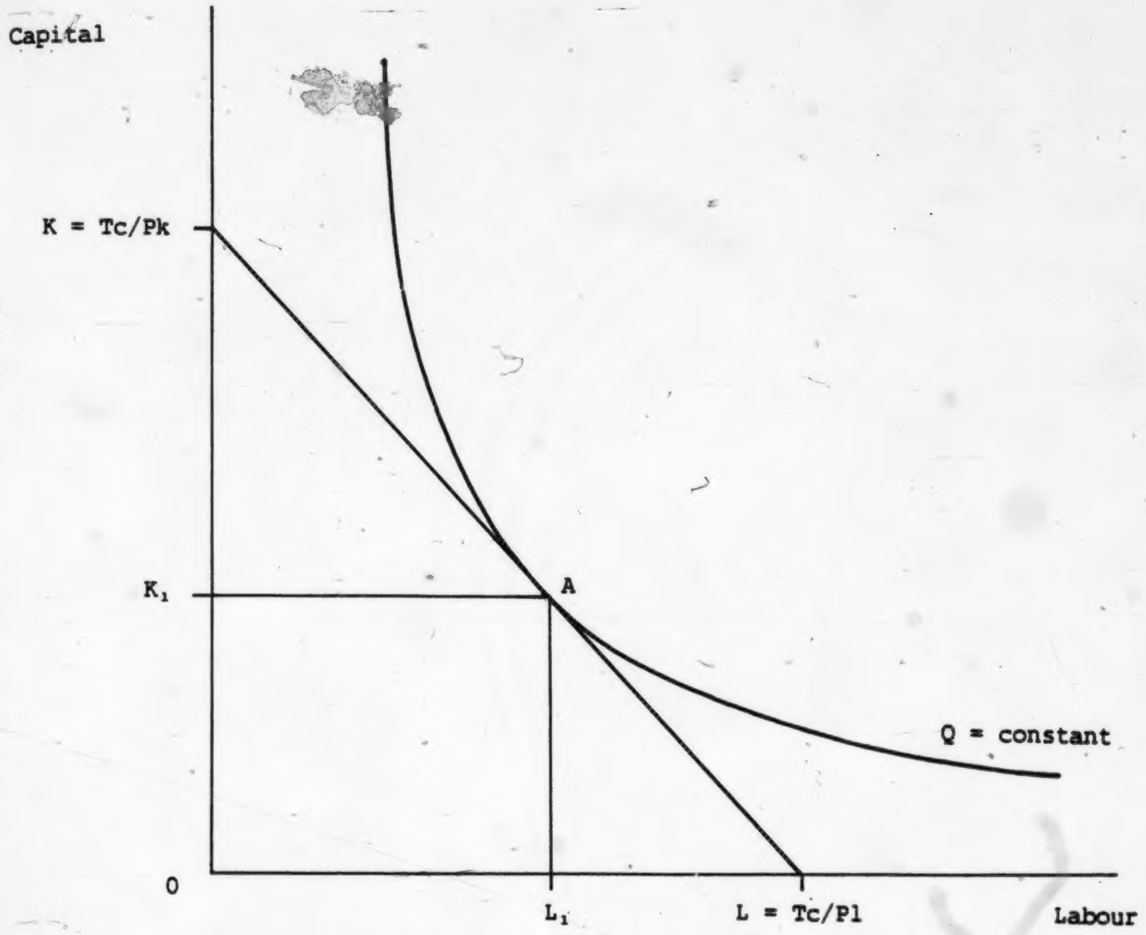

FIGURE 3.5. The Tangency Condition Between Isoquant and Isocost Curves

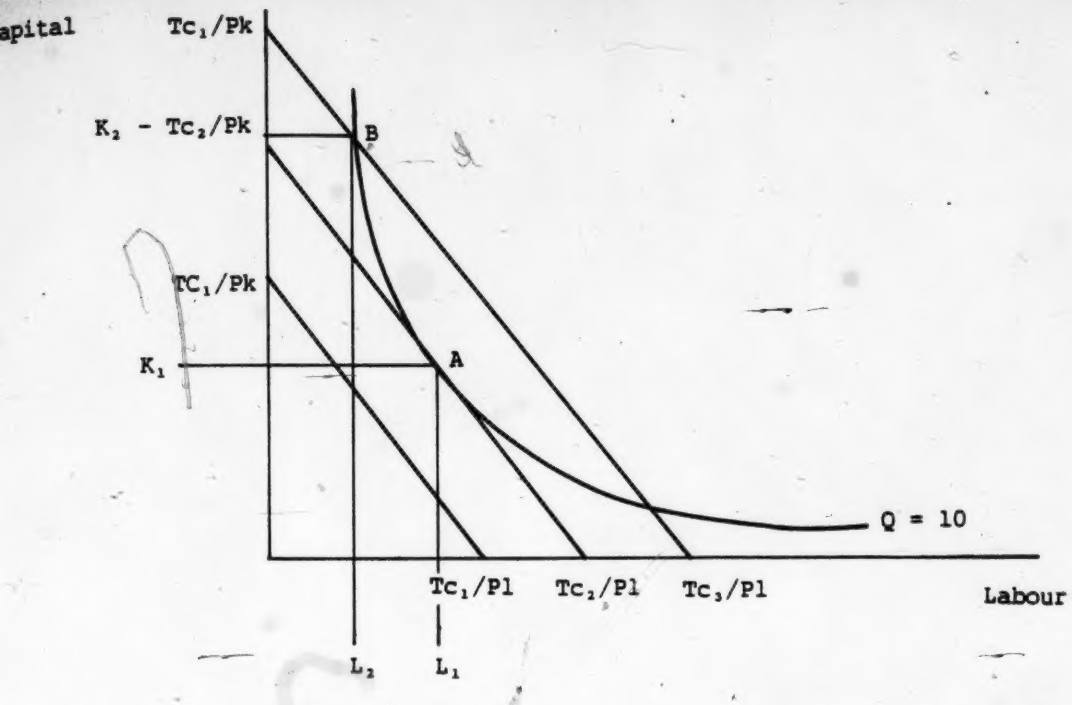

FIGURE 3.6. The Cost Minimizing Behaviour of the Profit Maximizing Firm

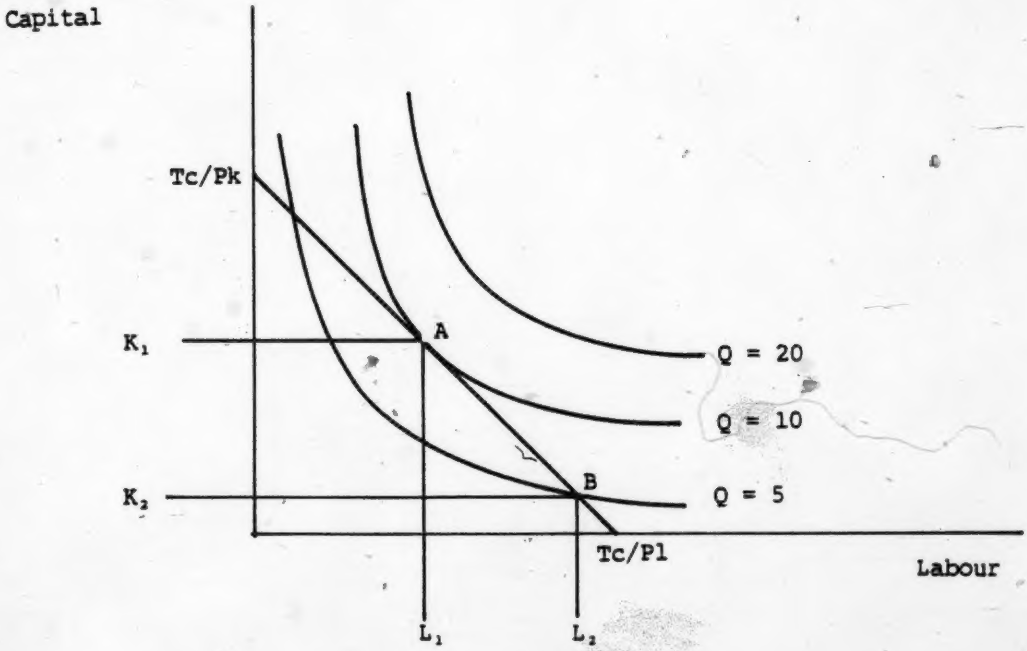

FIGURE 3.7. The Output Maximizing Behaviour of the Profit Maximizing Firm

have chosen K_2/L_2 at B for he would have exhausted his budget; however, the plant would only be producing half the potential output.

Clearly, no matter which motivating criteria is considered, the optimal input mix invariably occurs at the point of tangency between the isoquant and the isocost curves. Thus at the point of tangency the slopes of the two curves are exactly equivalent. Equation 3.5 showed that at any point on the isoquant the slope is defined by the MRTS which is equal to MPl/MPk. Equation 3.7 showed that the slope of the isocost line is a constant and is equal to Pl/Pk. Therefore we may conclude that the following equation represents the conditions required for choosing an optimal input mix:

Eq. 3.8 $Pl/Pk = MPl/MPk$

Thus the final choice of an input mix which is based on the technological limitations imposed by the isoquant and the economic limitations imposed by the isocost will satisfy both the necessary and sufficient conditions for optimal resource employment.

D. Facets of Production

To this point in the discussion of production theory the basic factor-product and factor-factor relationships have been discussed. Although the theoretical postulates are required for understanding firm behaviour in a theoretical sense, they lose pragmatic significance when observing the production relationships of real firms. In this endeavour an expanded approach is required. The expanded approach must allow for the effect of certain extraneous influences. The influences are termed returns to scale, capital intensity, technological variegation, and elasticity of substitution. The purpose of this section is to introduce

the theoretical and practical relevance of these supplemental facets of production.

Returns to Scale

Returns to scale are defined as the proportional change in output associated with a given change in inputs. For example, if all inputs are doubled and output is more than doubled then returns to scale are increasing. Alternatively, if, when inputs are doubled, output increases by an amount less than double, then returns to scale are decreasing. If a doubling of inputs results in a doubling of outputs, returns to scale are constant.

For illustrative purposes, increasing returns to scale are represented on Figure 3.8. Clearly a doubling of both capital and labour result in a greater than doubling of output from 100,000 fbm to 250,000 fbm. Increasing returns to scale occur because of the more efficient use of inputs caused by the technical, managerial, marketing and financial economies of scale[27] which are realized by larger operations.

Capital Intensity

The capital intensity of a production process is a relative measure of the amount of capital employed in relation to labour levels. Capital is an important element in the analysis of rationalized production because it has the ability to enhance the productivity of labour. The axioms developed earlier showed how the law of diminishing

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[27] Economies of scale are realized at the plant level through specialization or at the firm level via integration (horizontal, vertical, conglomerate). Diseconomies of scale result from limited management, bureaucracy, and increases in unit transportation costs (Shephard, 1979). This latter concept is discussed in more detail in Chapter VI.

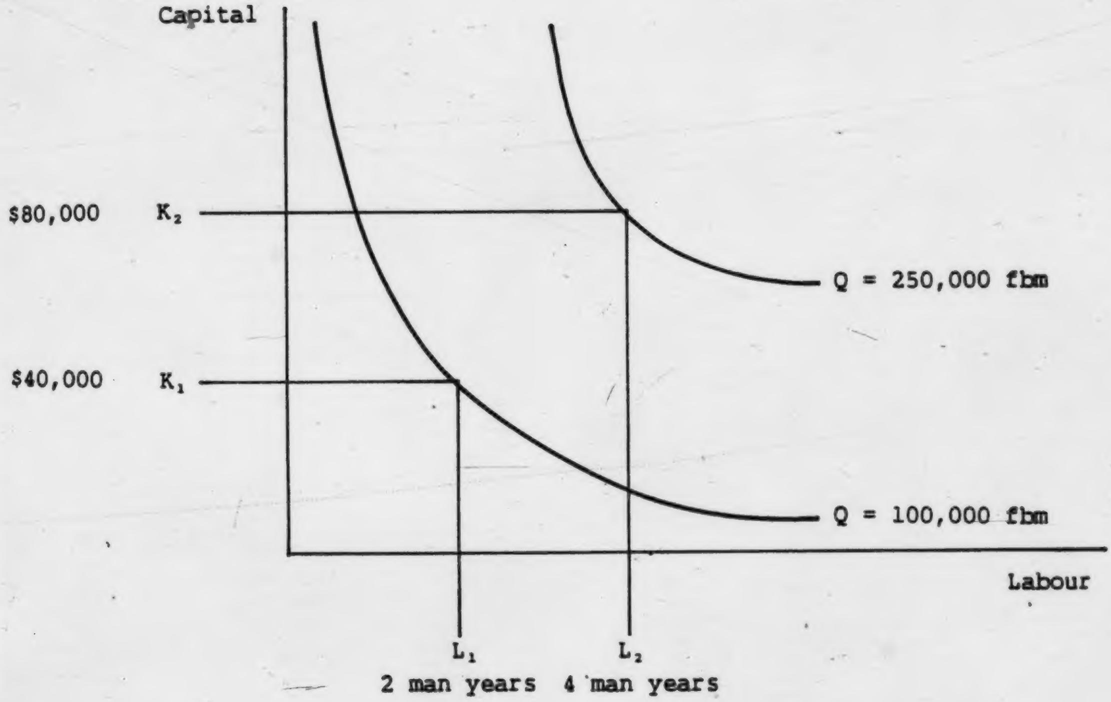

FIGURE 3.8. Increasing Returns to Scale For Sawmills

marginal productivity insured that as less labour is used in production then the marginal product of the last unit of labour is increased. It is sometimes beneficial then in times of rising labour costs to substitute capital for labour. Sandoe and Wayman (1977, p. 86) state

> "Lumbermen apparently improved their productivity by steadily reducing the work force while annual output remained the same. This was done because the pressures of rising wages and fringe benefits created an environment that favoured continuous substitution of machinery (capital) for labour."

Essentially then as the price of labour increases (relative to price of capital), the isocost line becomes steeper and the optimum mix of inputs is more capital intensive. Figure 3.9 illustrates this response. A firm producing a particular level of output and faced with a particular set of input prices (Pk_1, Pl_1) will initially employ K_1 and L_1 levels of capital and labour respectively. The exogenous impact of an increase in the price of labour however causes the isocost line to shift and K_1/L_1 is no longer optimal. Since the slope of the isocost line is defined by the ratio Pl/Pk a steeper line can be expected when labour wage increases. On Figure 3.9 the isocost line shifts from its original position (TC_1/Pk_1 - TC_1/Pl_1) to a steeper position (TC_1/Pk_1 - TC_1/Pl_2). Clearly the firm cannot produce the same level of output with the same total expenditure. The firm has two alternatives. It can reduce output and produce on a new isoquant with an input mix described by the tangency conditions of the new isocost line and the new isoquant. Alternatively, the firm can increase total expenditures so that previous levels of output can be maintained. Increased expenditures under the new price regime (Pk_1, Pl_2) cause a parallel shift of the isocost line. If

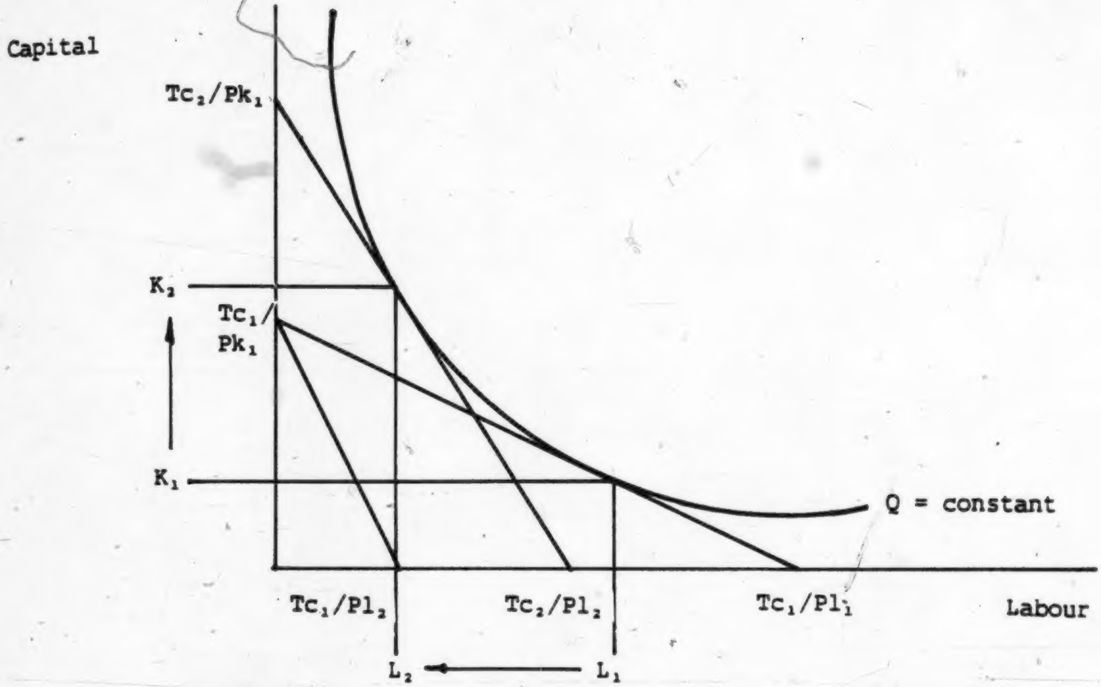

FIGURE 3.9. Capital Intensity of Sawmill Production Processes

X

the original output levels are to be maintained, the line must shift from the TC_1 position to the TC_2 position. The tangency point between the initial isoquant and the new isocost defines a more capital intensive optimal input mix (K_2/L_2). Since less labour is used in the production process, the marginal product (or productivity) of labour is enhanced.

Technological Variegation and Technical Efficiency

The discussion to this point has assumed a constant or static state of technology. In fact, technology is very diverse. Technology changes influence the productive capability of industries both through time and cross-sectionally. Because of this influence, a considerable difficulty exists in attempting to relate factor-output or factor-factor comparisons either for a cross-section of firms in an industry or for an entire industry over time. The production relationships of two firms producing the same output but by different procedures cannot be considered as the same because the firms are governed by structurally unique isoquants.

Figure 3.10 illustrates a particular form of technological variegation: neutral technological differences. The two isoquants in Figure 3.10 represent the same level of production; however isoquant $A'A'$ (which describes a recently developed processing technology) produces the output using less of both inputs. Thus the production method characterized by isoquant $A'A'$ is superior to AA. Since the MRTS at a particular K/L ratio is the same for both isoquants (implying no substitution of inputs) the technological variegation is classified as being of a neutral character. Specifically the variegation is classified

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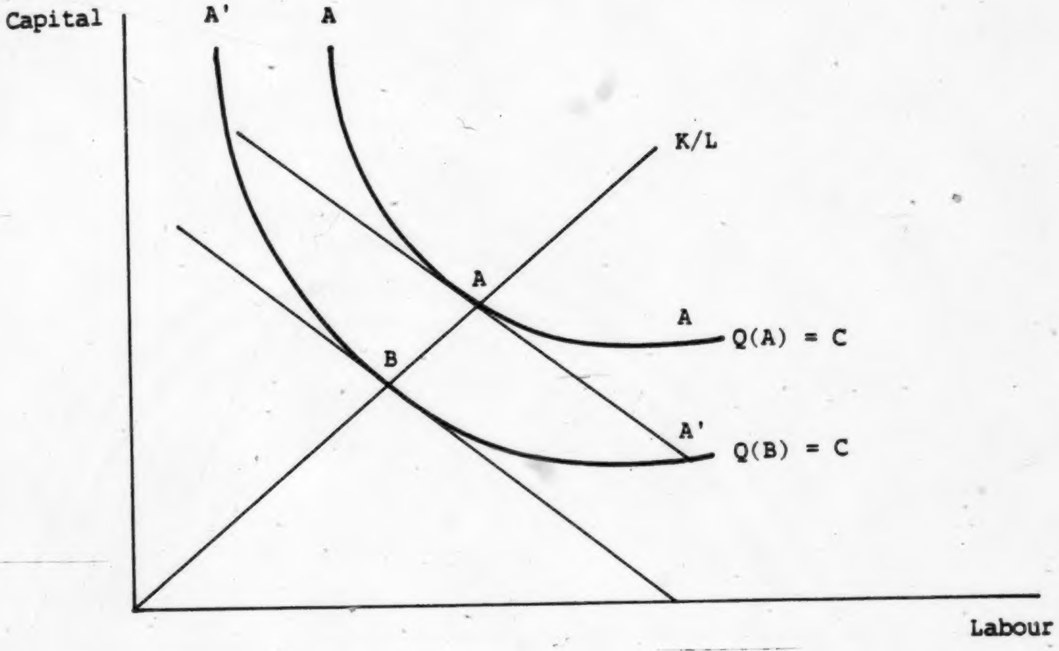


FIGURE 3.10. An Example of Hicks Neutral Technological Variegation

as "Hicks neutral" technological change.

Two other forms of technological differences include "Solow neutral" or "Harrod neutral" technological variegation. Implicit in these forms is the augmentation of the marginal products of a particular factor of production. In the Solow neutral case, the MP of capital is enhanced, resulting in a decline in the MRTS and a flattening of the isoquant curve. Since the utilization of this form of technology implies a substitution of capital for labour, it is termed labour-saving technological variegation.

Alternatively, if the MP of labour is enhanced then the technology of change is capital saving. This form of change is classified as "Harrod neutral" technological variegation.

The Solow and Harrod forms of technological variegation are represented on Figure 3.11. Figure 3.11 a) illustrates the labour-saving case. Isoquants Q(A) and Q(B) represent the same levels of output; however Q(B) exemplifies a more progressive technology. Because the MP of capital is increased, the isoquant becomes flatter. Assuming the input price ratio remains unchanged, the optimal input mix occurs at a higher position on the isoquant and the capital intensity of the production process increases.

Figure 3.11(b) illustrates the capital saving case. Since the MPl is enhanced, the isoquant becomes steeper. The optimal input mix becomes relatively more labour intensive.

Technological variegation can be further classified as being of two separate forms: biological and mechanical (Heady, 1952; Robinson, 1975). Biological differences are analogous to Hicks neutral technological changes because the marginal products of all factors are

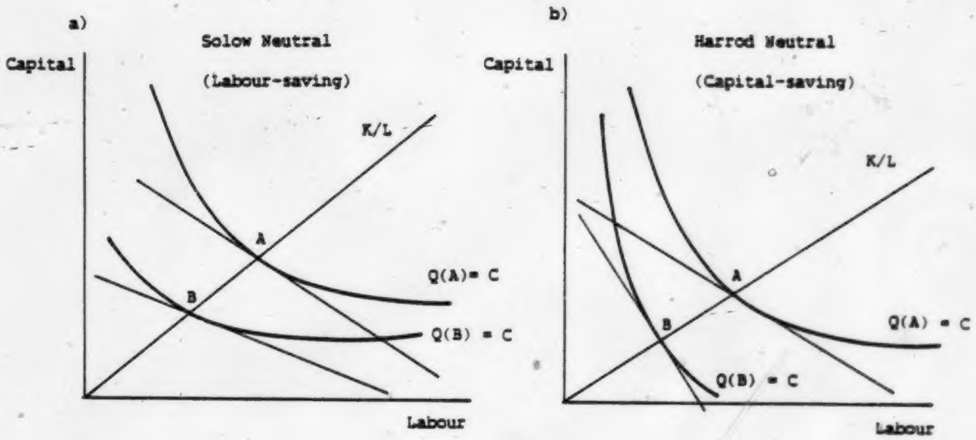


FIGURE 3.11. Examples of Solow and Harrod Neutral Technological Variegation

universally enhanced (e.g. thinner saw kerfs and enhanced utilization of conversion by-products (e.g. chips)). Mechanical differences on the other hand are labour-saving technological variegations the innovation of which is induced by higher labour costs (e.g. improvements in sorting and handling equipment, profile chip and canting equipment with computer assisted process control).

The differences in technology cross sectionally have important implications in terms of the industrial demand for two principal factors of production which are of particular interest to policy makers: stumpage and labour. In fact by the use of policy instruments such as quota allocation, input subsidization and public participation in research and development, technology can be managed to aid the achievement of particular socio-economic objectives. First the implications of particular forms of technology on the demand for labour and stumpage must be addressed. With specific regard to the effect of technological differences on demand for stumpage, Robinson (1975, p.153) states:

> "mechanical type innovations reduce cost; and hence tend to shift the product supply curve rightward. For a given demand schedule, this shift decreases the price of the product and increases the quantity demanded. Since the physical relationship between stumpage input and product output is not altered, mechanical innovations would tend to increase the demand for stumpage. Biological innovations on the other hand would tend to reduce this demand for stumpage since they increase the amount of product forthcoming from a given volume of stumpage."

In reference to the demand for labour Steir (1980, p. 473) states:

> "In a competitive industry, firms choose input levels such that the MRTS equals the factor price ratio. Since the technological change bias is defined for a constant capital-labor ratio, a labor-saving bias (from mechanical innovation) would cause the demand and income share for labor to decline."

Thus mechanical innovations have a positive influence on relative cost competitiveness and stumpage demand and a negative influence on the relative demand for labour. Biological innovations, on the other hand, influence stumpage requirements negatively and have no direct affect on labour demand.

The form of cross-sectional technological variegation, be it biased or neutral, has very important implications in terms of certain policy objectives and the procedures for achieving them.

The foregoing discussion of technological variegation shows how production relationships change with qualitative differences in the variable factors of production (capital, labour). Technical efficiency is an additional way of describing inter-firm differences in total factor productivity. Yotopolous and Nugent (1976, pg. 74) describe technical efficiency with "firms have different input-output mixes because they have different endowments of fixed (e.g. management) factors of production, that is, they have neutral differences in technical efficiency." In the case of cross-sectional data the fixed factor may be entrepeneurial or managerial capability. Thus, measured differences in output produced by two firms using the same bundle of homogeneous inputs (therefore the inherent production function is the same) is attributable to the fact that one firm has better technical knowledge and is therefore technically more efficient. The difference

between technological variegation and technical efficiency is that the former accounts for qualitative differences in the inputs whereas the latter measures the contribution of entrepeneurial capability toward more effectively combining homogeneous inputs. Thus whereas technological variegation is reflected in structurally unique production functions, technical efficiency assumes the presence of one isoquant (or one production function). Clearly in the sawmill industry, where individual firms utilize a wide array of production equipment and retain different endowments of managerial capability, both technological variegation and technical efficiency will be reflected.

Elasticity of Substitution

The final facet of the firm's technology is the "elasticity of substitution". This concept is an important and relevant aspect of the production technology of the firm.

The elasticity of substitution (σ) is defined as the percentage change in the capital labour ratio given a 1 percent change in the marginal rate of technical substitution (MRTS). Algebraically the elasticity of substitution is equivalent to

Eq. 3.9 $$\sigma = \frac{\Delta(K/L)/(K/L)}{[\Delta(MPl/MPk)]/[MPl/MPk]}$$

The elasticity of substitution is basically a measure of the ease with which one factor can be substituted for another factor along the isoquant. A large σ suggests that factor inputs are easily substitutable and that there is very little curvature in the isoquants. Thus a 1%Δ in MRTS causes relatively large Δ in K/L. Alternatively a low σ suggests that inputs are not easily substituted and that the isoquant has a large curvature. Thus a 1% Δ in MRTS causes a relatively smaller Δ in K/L.

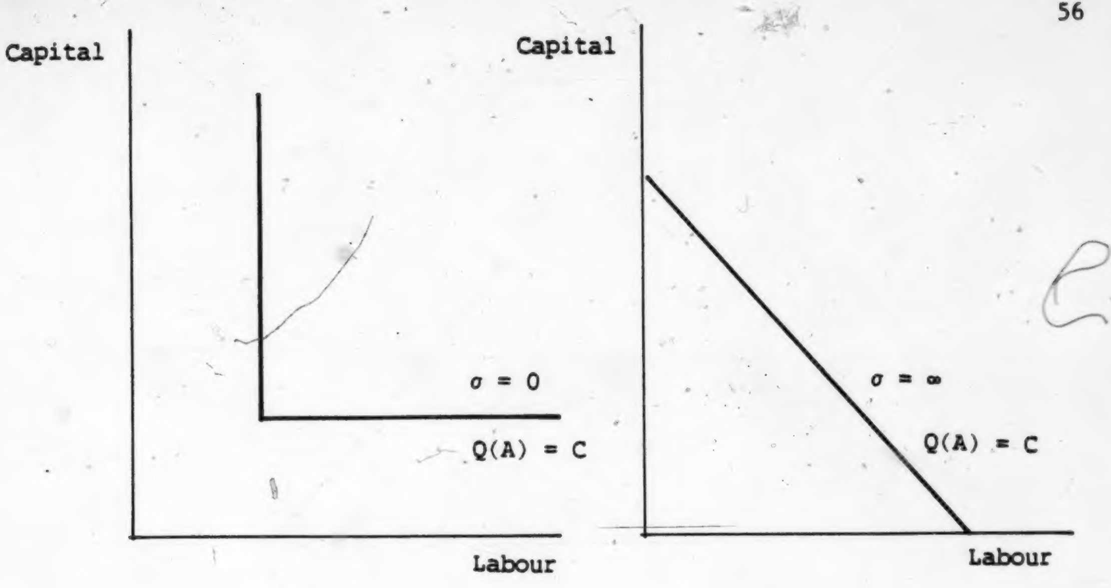
Figure 3.12(a) represents the extreme values which σ can take. The first assumption represents the Leontiff case of no possibilities of substitution. A 1% change in MRTS causes a 0% change in factor proportions. Only one combination is possible. The second case represents the opposite situation of perfect substitutability. Here substitution of one input for another has no effect on the MP of the factors and a single optimum combination of inputs is non-existent. Production will proceed with all of a particular factor of production being used. The choice of a particular factor will depend on which one is cheapest on a per unit basis. Figure 3.12(b) represents the more general cases of curved isoquants. The elasticity of substitution is enlightening in these cases in that the degree of curvature of the isoquants can be ascertained by the magnitude of the value of σ. The $\sigma(Y)$ for $Q(A) = B$ is less than $\sigma(X)$ for $Q(A) = C$. Thus substitution is easier for isoquant $Q(A) = C$ than for $Q(A) = B$.

An important application of the elasticity of substitution concept is to assess the effect of changes in the price of a factor of production on the firm's costs of production. As Quirk (1976, p. 129) states

> "The impact that a change in the wage/rental ratio has on marginal cost of the firm depends on the ease with which factors can be substituted for one another. In general, the more easily factors can be substituted,the less will a change in the wage/rental ratio change the marginal cost of the firm. An increase in the price of labour, for example, can be partly offset by a shift to more capital-intensive processes."

Equation 3.9 described the σ in terms of the response of factor

a) Extreme Values



b) General Case

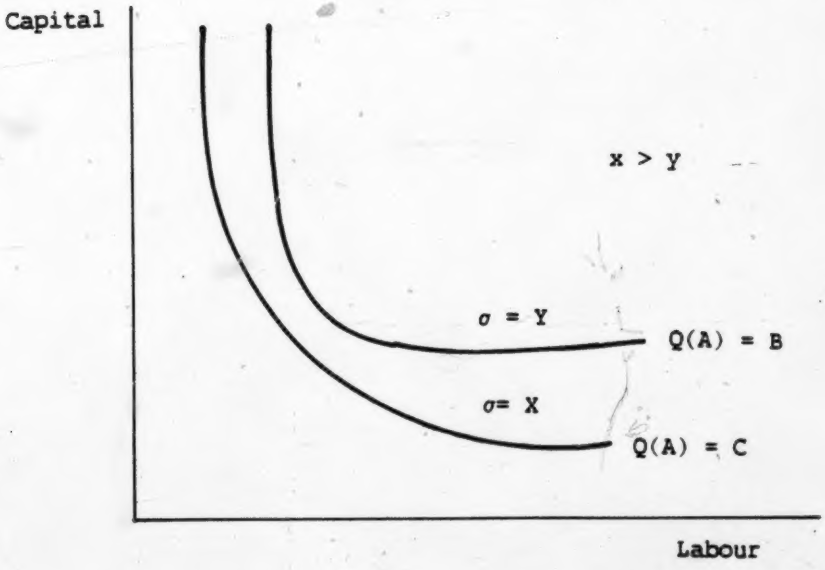


FIGURE 3.12. Elasticities of Substitution

proportions to a 1% change in MRTS. From Equation 3.8, however, we also know that MRTS = Pl/Pk. Therefore the denominator of Eq. 3.9 can be replaced with Δ log Pl/Pk to form;

Eq. 3.10 $$\sigma = \frac{\Delta(K/L)/(K/L)}{\Delta(Pl/Pk)/(Pl/Pk)}$$

Thus, in Equation 3.10 the elasticity of substitution indicates the percent change in factor proportions associated with a 1% change in the ratio of input prices. In that certain forms of government subsidies can alter the effective input price ratio, the magnitude of σ can be of considerable significance to policy makers.

E. The Theoretical Model of Perfect Competition

Since a basic underlying assumption of the models developed in the next chapter is that firms operate in perfectly competitive input and output markets, a description of the nature of these markets and of the behaviour of firms within this environment is appropriate. The perfectly competitive market model is characterized by four features (Thompson 1973); 1) products are undifferentiated and buyers pay the same price for the product irrelevant of who produces it, 2) individual buyers and sellers cannot influence the prices of products or factors by their procurement decision and therefore, firms are price takers in both product and factor markets, 3) resource inputs are completely mobile and firms are free to enter or leave the industry, and 4) there is perfect and complete knowledge by producers and consumers and all production and consumption decisions are made with complete certainty of the outcome. Economic theory suggests that under the influence of these four features, firms will behave in some predictable manner. Two separate time frames are considered: the short run and the long run. In the short

run, the business decisions of the firm are constrained by the fixicity of the capital stock and by the feature that output prices are constant at all levels of production (i.e. the demand curve observed by the firm is completely in elastic). Under this scenario, the firms managers have basically only one decision variable. By adjusting the level of use of the variable input the firm can increase or decrease output. Assuming the objective of the competitive firm is to maximize profits, the firm will produce at a level where marginal costs equals marginal revenue (MR = AR = price of output = MC). This situation is illustrated in Figure 3.13. At a given market derived output price of p_1, the firm will produce Q* units of output. Here profits are maximized and are indicated by the hatched area on Figure 3.13.

In the longer run time frame, firms are not limited by a fixed capital stock. Thus for industries where profits are being realized (as depicted in Figure 3.13) new firms will enter and existing firms will expand. The consequent greater supply of output pushes the market derived product price downward by shifting the industries supply curve rightward. If output price moves too far down, economic losses will occur and firms will depart the industry causing output price to increase (because of a leftward shift in the supply curve). The long run equilibrium position of a firm operating in perfect competition is indicated in Figure 3.14. The position of long run equilibrium is at Q* where; price equals marginal revenue equals short run average cost equals the minimum of the long run average cost curve. At any position other than at Q*, the occurrence of economic profits or losses will cause the "forces of adjustment to spring into motion". The stable industry in long run equilibrium will be characterized by a large number

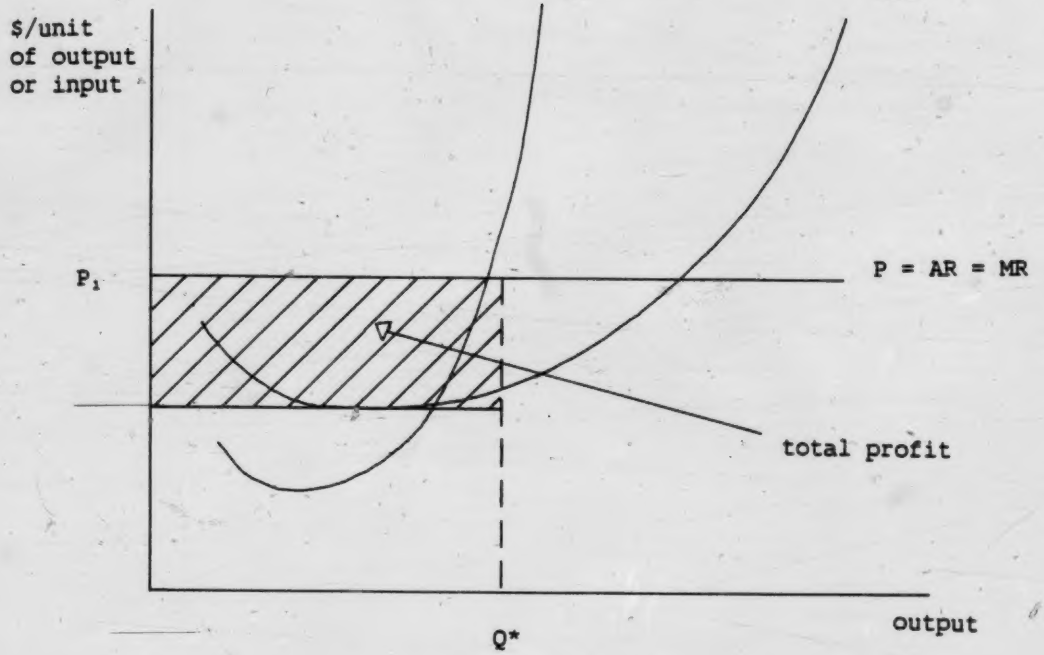


FIGURE 3.13. Short Run Equilibrium of A Perfectly Competitive Firm

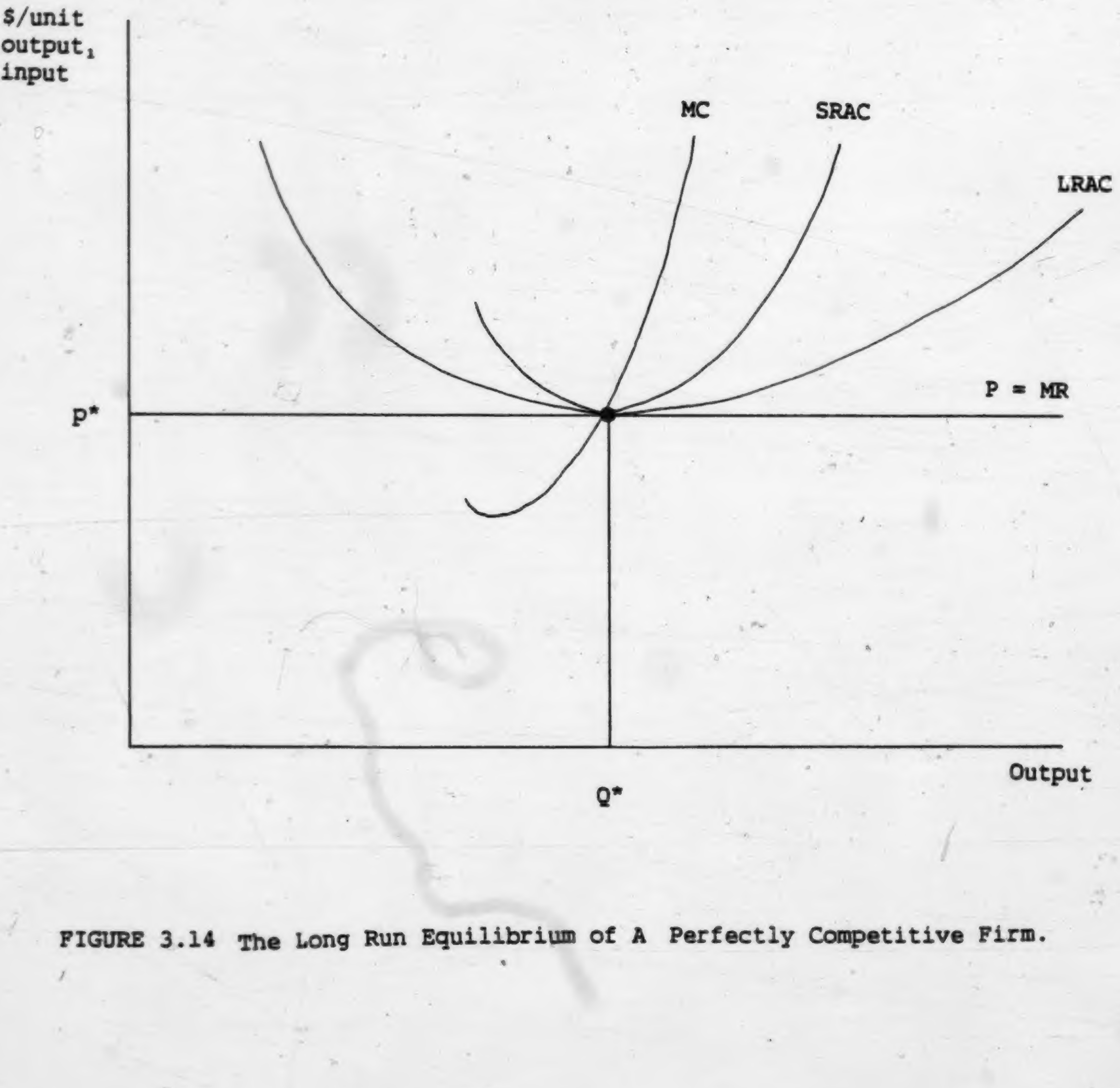


FIGURE 3.14 The Long Run Equilibrium of A Perfectly Competitive Firm.

of firms of exactly equal size producing an undifferentiated product by exactly the same technologies and using the exactly same qualities and quantities of factors of production. Factor and product prices are given and are the same for all firms in the industry. Since the firm is at the minimum of the LRAC curve, economies or diseconomies of scale are non-existant (which is synonomous to constant returns to scale).

With time series data, the elasticity of substitution measures substitution responses caused by factor price changes over time. In a cross-section, the concept is somewhat more nebulous. In the cross-section, used in this thesis the elasticity is based on observed intra-industry differences in factor proportions resulting from intra-industry variation in factor prices. As previously described in this section, the model of perfect competition states that factor prices must be uniform across the particular industry. This imposition would appear to cause an anomoly in the methodology employed in this thesis in that factor substitution would not be measurable (assuming the axiom is valid). For the purposes of this thesis however the restrictive condition of uniform input prices is relaxed and the concept of an elasticity of substitution is considered to be valid. This approach conforms to many studies which either estimate elasticities of substitution directly (e.g. Arrow, Chenery, Minhas, Solow, 1961; Griliches, Ringstad, 1971) with cross-sectional intra-industry data or implicitly assume substitutability between inputs in a cross-sectional (Yotopoulos, Lau, Lin, 1976; Sidhu and Baanante, 1981).

F. The Production Function

The production function is the means by which quantitative significance is afforded the relationships described in Section D. Given the availability of information regarding levels of factor use, unit prices of factors of production, output levels and unit price of output, an array of production variables can be ascertained via the production function.

Mansfield (1975, p. 122) defines the production function with

> "The production function is the relationship between the quantities of various inputs used per period of time and the maximum quantity of the commodity that can be produced per period of time.... The production function summarizes the characteristics of existing technology at a given point in time; it shows the technological constraints that the firm must reckon with."

As described in Chapter One, two explicit functional forms are employed in this thesis. The Cobb-Douglas production function (Eq. 1.2) is represented by

Eq. 3.11 $$Y = A \cdot K^{(1-\alpha)} \cdot L^{\alpha}$$

where: Y = sawmill output

A = technical efficiency parameter

K = capital input

L = labour input

α= production elasticity of labour

$(1 - \alpha)$ = production elasticity of capital

This function, which was popularized in 1928, was the first major development in the evolution of neoclassical production economics. The

function is based on C.W. Douglas's original observation that the total amount paid to labour in a production process is some constant proportion of total output or that;

Eq. 3.12 $\omega \cdot L = \alpha Y$ or $\omega = \alpha Y/L$

where: ω = labour wage rate

L = amount of labour

Y = total output

α = a production parameter.

Since the marginal product of labour is equal to ω, Eq. 3.12 is transformed to;

Eq. 3.13 $\partial Y/\partial L = \alpha \frac{Y}{L}$

The contribution of Cobb was to suggest that the integrated form of Eq. 3.13 was the same as equation 3.11. Differentiating Eq. 3.11 with respect to labour provides;

Eq. 3.14

$$\begin{aligned} \partial Y/\partial L &= A \cdot K^{(1-\alpha)} \alpha \cdot L^{(\alpha-1)} \\ &= (A \cdot K^{(1-\alpha} \cdot L) \frac{\alpha}{L} \\ &= Y \cdot \frac{\alpha}{L} = \alpha \cdot \frac{Y}{L} && = \text{Eq. } 3.13 \end{aligned}$$

Thus, Cobb's proposed equation accurately represented Douglas's empirical observations.

A more generalized form of Eq. 3.11 is;

Eq. 3.15 $Y = A \cdot K^{\beta} \cdot L^{\alpha}$

This equation is more flexible because the function is not constrained to linear homogeneity (i.e. the sum of the production elasticities are not required to sum to unity).

Although the Cobb-Douglas function can be generalized so that linear homogeneity is not a constraint, the function is still limited in

that the implicit elasticity of substitution is constant and is always equal to one. This limitation is overcome in the "Constant Elasticity of Substitution" production function the explicit form of which is represented by;

Eq. 3.16 $Y = \gamma[\delta \cdot K^{-\rho} + (1-\delta)L^{-\rho}]^{-\tau/\rho}$

where Y = sawmill output

K = capital input

L = labour input

γ = technical efficiency parameter

δ = capital intensity parameter

ρ = substitution parameter

τ = returns to scale parameter

This function was the second major development in neoclassical production functions and was popularized by Arrow et al. in 1961. The function is generalized from the Cobb-Douglas function by providing for a non-unitary elasticity of substitution. Heathfield (1971) shows that whereas the Cobb-Douglas function is based on;

Eq. 3.17 $Y/L = \alpha \cdot \omega$

the CES function is based on

Eq. 3.18 $Y/L = \alpha \cdot \omega^{\sigma}$

where σ = elasticity of substitution

and α in 3.17 is the inverse of α in 3.18.

From a purely theoretical standpoint the concept of a measurable production function within a cross-sectional framework is a violation of the theoretical model of a firm operating within a purely competitive market. Yotopoulos and Nugent (1976, pg. 74) state: "The attempt to quantify economic efficiency through output-input and input-input ratios

constitutes measurement without theory ... Economic theory specifies the conditions under which firms are expected to have identical ratios of inputs and outputs. Specifically, it is well known that all firms would have the same quantities of inputs and outputs (and as a result only one point on the production surface would be observable) if:

1. All firms had the same production function, that is, the same technical knowledge and identical fixed factors;
2. All firms faced the same prices in the product and factor markets;
3. All firms maximized profits perfectly and instantaneously."

Observable points in the input-output space not at the single point of economic efficiency would simply be attributable to measurement errors. Accounting for all differences as simply measurement error is an incorrect approach. In relaxing the constraints of perfect competition we acknowledge the appropriateness of the production concepts and the measurability of structurally unique production functions with a cross-section of firms. For example, simple observation shows that firms in the sawmill industry operate with a wide range of equipment types and therefore they are not represented by the same production function. Closer observation shows that in fact firms do not face the same prices for factors of production providing credibility to the concept of measurable substitutability between inputs. Intuitive speculation suggests that firms do not all maximize profits "perfectly and instantaneously" and therefore firms may differ strictly by the capability of their managers. Thus in comparing the behaviour of firms in a real industry (such as the sawmill industry) with the behaviour of the firm as suggested by the perfectly competitive model, the fragility of the model's axioms become evident. As Thompson (1973 pg. 342) states:

"Obviously these four conditions (of perfect competition) are so stringent that no market in the real world ever has or ever can meet them ... Nonetheless, the study of perfectly competitive markets is not without value ... a model may yield valid conclusions even though its assumptions are "unrealistic" ... the perfectly competitive model characterizes fairly well the behaviour of owner-managed, periphery firms in certain market circumstances ... we shall, however, be careful not to claim too much for the model of perfect competition, and we shall be especially judicious in applying its conclusions to the behaviour of firms in actual situations."

The existance of differential labour wages suggests the possibility of segmented labour markets for the Alberta sawmill industry. In fact, in observing variations in size across the sawmill industry there appears to be some correlation between the size of the community and size of the mill with larger mills being located in larger towns. To some extent this feature may explain why higher labour wages are paid in the larger mills. Labour wage differentials observed by the Alberta sawmill industry are permanent equilibrium differences in the labour market caused by an immobility of labour which prevents labours wage from being equalized. Another reason for the existance of persistent differentials is that the sawmill labour force is only partly unionized.

G. Summary

This chapter has described some of the principle theoretical economic postulates required for understanding production relationships. In Section B, the law of diminishing marginal product and the three

stages of production were defined. In Section C, factor-factor production relationships were emphasized and the necessary and sufficient conditions for optimum input use were outlined. In Section D, the appropriateness of the production concepts of returns to scale, capital intensity, technological variegations and elasticity of substitution was elucidated. In Section E the perfectly competitive model was introduced. In the last Section, the concept of the production function was introduced and two explicit functional forms were described.

The theoretical constructs provided in this chapter serve to provide a framework from which the reasons why resources are allocated as they are by the firm can be comprehended. The following chapter builds upon this framework by describing specific methodologies which provide for quantification of the theoretical concepts.

IV. ANALYTIC FRAMEWORK FOR EMPIRICALLY ANALYZING PRODUCTION IN THE ALBERTA SAWMILL INDUSTRY

A. Introduction

The purpose of this chapter is to develop the models, describe the techniques for estimating the parameters of the models, and to characterize the properties of the estimates. The chapter comprises two major sections: methodologies and procedure. In the first section, two specific economic models are developed. The first model is Solow's model for measuring technological change. The foundation of this model is the familiar neo-classical production function known as the Cobb-Douglas function. The second model is based on the second major development in the Marshallian line of production functions (Heathfield, 1971). The function is known as the Constant Elasticity of Substitution (CES) function. In the second section (procedures) the estimation techniques and statistical properties of the estimates of both models are discussed.

B. Methodologies

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The Solow Model

The first model developed in this section is one originated by R. M. Solow (Solow, 1957). His intention was "to describe an elementary way of segregating variations in output per head due to technical change from those due to changes in the availability of capital per head" (Solow, p. 312). The model provides a convenient and simple way of separating movements along the production function (increased output

resulting from employment of more capital intensive production processes) from total shifts (resulting from technological change) in the production function.

The model, as originally designed, was dynamic and was oriented toward allowing for the effects of changing capital over time. The approach was to incorporate a time variable (t) as an argument in the implicit production function thereby implying the effects on productivity of innovation of new technologies through the years. This relationship is shown in Equation 4.1.

Eq. 4.1 $Y = f(K, L, t)$

where Y = total output

K = capital input

L = labour input

t = a time variable

In Chapter II a wide array of methods of producing lumber in Alberta was described in some detail. Some sawmills employ the more traditional circular headsaws and other mills utilize more recently developed methods of production.[28] In general, the small sawmills employ the traditional (and what would intuitively be expected to be less efficient) methods of production. As mentioned in Chapter III, this variability in size of plant and methods of production is a contradiction to the perfectly competitive model. In fact, firms are not exactly the same size (as expected with the theoretical model) but vary widely. Also, firms do not utilize the same technology but use a wide range of production methods. We may therefore conclude that Alberta sawmills do not conform to the behaviour of firms in the perfectly

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[28] Such as Chip-N-Saws, gang saws, by-product recovery equipment, and other machinery designed for high speed linear throughput.

competitive model in all respects. They are, in fact, not faced by one unique production function but moreover by a range of functions described by different scalars, slopes and elasticities. We must however retain the perfectly competitive model assumptions of firms being price takers and that plant managers endeavor to and actually do maximize profits, in order to be able to measure the different production frontiers. Thus, assuming that all firms are price efficient (that is they equate factor price ratios to the MRTS), cross-sectional differences in the parameters of production can be attributed to either technological or managerial differences between firms.

A methodological question that becomes evident at this point is: How can the cross-sectional data be organized to systematically reflect technological or managerial differences in the industry? A possible answer is to rank the firms according to size. From general knowledge of the sawmill industry, we know that technology in the largest mills is generally "state of the art" whereas technology in the smaller mills is somewhat less progressive (see Chapter III). We may also suspect that management in larger firms is more adept than in smaller firms. By organizing the cross section according to size of firm we impose a structure to reflect what is suspected to be qualitative differences in factors of production.

The Solow model appears well suited to analyzing the potential productivity effects of differences in technologies in the Alberta sawmill industry cross-sectionally. A modification of the original form (Eq. 4.1), however, is necessary to transform the model from a dynamic one to one that can be applied to the spectrum of Alberta sawmills at one point in time. The following implicit form is proposed:

Eq. 4.2 $y = f(K, L, s)$

where y, K and L are the same as in Eq. 4.1 but the variable (t) is replaced by a size class variable (s) to separate differences in techology from capital deepening across differing size classes of firms.

Before the model can be further established, a number of cases underlying the model are described. The assumptions include:

1. Technical variegation is product augmenting or "Hicks" neutral. (Therefore marginal rates of technical substitution remain unchanged in comparing different technologies).
2. There exists perfect competition in all factor markets and factors of production are paid at a rate equal to their marginal products.
3. The production function is linearly homogeneous and therefore exhibits constant returns to scale (as in the original Cobb-Douglas functional form).
4. The elasticity of substitution between capital and labour is equal to unity (as implied by the Cobb-Douglas functional form).

The adoption of these simplifying assumptions facilitates a rather straightforward approach to measuring neutral technical variegation. There is, however, an opportunity cost of simplicity. The assumptions, so employed, allow the analyst to abstract from reality and it could be argued that the abstractions are unrealistic enough to invalidate the model by overtly biasing the estimates. The extent to which this might be true is discerned in the chapter VII.

Based on the first assumption the generalized functional form (Eq. 4.2) is forced into the form:

Eq. 4.3 $Y = A(s)\, f(K, L)$

Here $A(s)$ enters the function multiplicatively and measures the

accumulated shifts in the function resulting from product augmenting technological variegation[29] across separate sizes of operations. The explicit composite is expressed in the recognizable linearly homogeneous Cobb-Douglas format.

Eq. 4.4 $$Y = A(s)\, L^{1-T} \cdot K^{T}$$

where $$T = \frac{I \cdot K}{P \cdot Y}$$

I = cost per unit of capital

P = price per unit of output

Y = number of units of output

Dividing both sides of equation 4.4 by L (to account for changes in size of the labour force) yields the following relationship:

Eq. 4.5 $$Y/L = A(s)\,(K/L)^{T}$$

Conversion to log form and differentiating provides:

Eq. 4.6 $$d \log Y/L = d \log A(s) + T \cdot d \log (K/L)$$

or

Eq. 4.7 $$\frac{d\,Y/L}{Y/L} = \frac{d\,A(s)}{A(s)} + T\,\frac{d(K/L)}{K/L}$$

which in discrete terms is equivalent to

Eq. 4.8 $$\frac{\Delta A(s)}{A(s)} = \frac{\Delta Y/L}{Y/L} - T \cdot \frac{\Delta K/L}{K/L}$$

Now with scale series data on output per unit of labour (Y/L), capital per unit of labour (K/L) and capitals share of output (T), $\Delta A(s)/A(s)$ (which represents the % change in production resulting from changing technology between two sizes of firm) can be easily derived.

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[29] There is a problem in separability here in that the portion of A(s) attributable to technological differences in processing methods (technological variegation) and the portion attributable to the achievement of managers (technical efficiency) is not readily identifiable. This is an empirical question which remains unanswered. For this portion of the study all measurable shifts in the production function are termed neutral technological variegation.

With the calculated values of $\Delta A(s)/A(s)$, an index of technical variegation can be ascertained across the pre-determined scales of production. By setting the initial scale A(s) equal to one, subsequent A(s)'s are derived according to:

Eq. 4.9 $$A(s+1) = A(s) \cdot (1 + \Delta A(s)/A(s))$$

With calculated values of A(s) for each scale of production, the aggregate cross-sectional production function for capital, both with and without the effects of technological differences removed, can be investigated. In the Cobb-Douglas case, the aggregate production function is represented by

Eq. 4.10 $$\log Y/L = B_o + B_1 \log K/L$$ [30]

The aggregate production function, adjusted for the influences of technical variegation, is represented by:

Eq. 4.11 $$\log [(Y/L)/(A(s))] = B_o + B_1 \log K/L$$

Equation 4.11 shows the effect of output per unit of labour of pure capital deepening and equation 4.10 describes the effect on output per unit of labour of technological variegation and capital deepening combined.

The Explicit C.E.S. Production Function

The purpose of the model developed in this section is to provide an initial aggregate estimate of the parameters underlying production in the Alberta sawmill industry. Specifically the intent is to provide estimates of;

1. the elasticity of substitution between capital and labour,
2. the capital intensity of production processes,

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[30] Appendix A shows by Eulers Theorem that under constant returns to scale $[Y = f(K,L)] \simeq [Y/L = f(K/L,1)]$.

5.

3. the level of returns to scale in the industry, and
4. the nature of technological variegation

The explicit functional form underlying the model is the C.E.S. production function.

The C.E.S. functional form is superior to the previous Cobb-Douglas form for two reasons:

1. The underlying production function is not limited by forcing it to be linear homogeneous.
2. The elasticity of substitution is not restricted to unity.

The required assumptions embodying this model include:

1. Technical variegation is exogenous or disembodied or 'Hicks' neutral.
2. Firms operate in perfectly competitive input and output markets and factors of production are paid according to each specific input's marginal productivity.
3. The direct elasticity of substitution is constant at all factor price ratios.

The explicit form of the C.E.S. function is as follows:

Eq. 4.12 $Y = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\tau/\rho}$

where Y = output
K = capital
L = labour
γ = technical efficiency parameter
δ = capital intensity parameter
ρ = substitution parameter
τ = returns to scale parameter

The following restrictions are imposed on the values of the

parameters:

$$-1 < \rho < \infty$$

$$\rho \neq 0$$

$$0 < \delta < 1$$

$$\gamma , \tau > 0$$

The problem with Eq. 4.12 in terms of its direct use as an economic model for characterizing production technology is that it is an intrinsically non-linear function and direct linear estimation of the parameters is impossible. Sequential estimation of the parameters, however, can be achieved by separating the non-linear function into two separate intrinsically linear forms.

Recall from Chapter 3 that production theory states that the ratio of the marginal products of factor inputs is equal to the marginal rate of technical substitution (MRTS). In the context of the C.E.S. function the MRTS is derived by first partial-differentiating Eq. 4.12 with respect to labour and dividing by the first partial derivative of capital. First, Equation 4.12 is rewritten to:

Eq. 4.13 $$Y^{-\rho/\tau} = \gamma^{-\rho/\tau}[\delta K^{-\rho} + (1-\delta)L^{-\rho}]$$

Partially differentiating with respect to labour provides:

Eq. 4.14 $$-\rho/\tau^{(-\rho/\tau)-1} \frac{\partial Y}{\partial L} = -\rho \gamma^{-\rho/\tau} (1-\delta)L^{-\rho-1}$$

∴ Eq. 4.15 $$MPL = \frac{\partial Y}{\partial L} = \tau(1-\delta)\gamma^{-\rho/\tau} \cdot \frac{Y^{1+\rho/\tau}}{L^{1+\rho}}$$

Partially differentiating with respect to capital provides:

Eq. 4.16 $$-\rho/\tau \; Y^{-\rho/\tau-1} \frac{\partial Y}{\partial K} = -\rho\gamma^{-\rho/\tau} \delta K^{-\rho-1}$$

or

Eq. 4.17 $$MPK = \frac{\partial Y}{\partial K} = \tau\delta\gamma^{-\rho/\tau} \cdot \frac{Y^{1+\rho/\tau}}{K^{1+\rho}}$$

Thus

Eq. 4.18 $$MRTS = \frac{MPL}{MPK} = \frac{\tau(1-\delta)\gamma^{-\rho/\tau}\ Y^{1+\rho/\tau}\ L^{(1+\rho)^{-1}}}{\tau\delta\gamma^{-\rho/\tau}\ Y^{1+\rho/\tau}\ K^{(1+\rho)^{-1}}}$$

Equation 4.18 reduces to:

Eq. 4.19 $$MRTS = \frac{1-\delta}{\delta}\left(\frac{K}{L}\right)^{1+\rho}$$

or

Eq. 4.20 $$\frac{K}{L} = \left(\frac{\delta}{1-\delta}\right)^{1/(1+\rho)} \cdot MRTS^{1/(1+\rho)}$$

In log form the relationship is represented by:

Eq. 4.21 $$\log\frac{K}{L} = \left(\frac{1}{1+\rho}\right)\log\left(\frac{\delta}{1-\delta}\right) + \left(\frac{1}{1+\rho}\right)\log MRTS$$

Layard and Walters (1978, p. 266) define the direct[31] elasticity of substitution as "the proportional change in K/L associated with a unit proportional change in fL/fK (or MRTS) holding output constant."

Algebraically the direct elasticity of substitution is defined as:

Eq. 4.22 $$\sigma = \frac{\partial \log (K/L)}{\partial \log MRTS}$$

Partially differentiating equation 4.21 with respect to the MRTS provides:

Eq. 4.23 $$\frac{\partial \log (K/L)}{\partial \log MRTS} = \sigma = \frac{1}{1+\rho}$$

Thus the direct elasticity of substitution term adjusts the production relationships for biases that would be introduced to the

---

[31] It should be noted that there is a conceptual difference between 'direct' elasticity of substitution and 'Allen' partial elasticities of substitution.

function in situations where the substitution parameter is unequal to zero (or equivalently where $\sigma \neq 1$).

Replacing $1/1+\rho$ in Eq. 4.21 with σ provides:

Eq. 4.24 $$\log \frac{K}{L} = \sigma \log \frac{\delta}{1-\delta} + \sigma \log \text{MRTS}$$

Recall that a required assumption states that price taking, profit maximizing firms operating within perfectly competitive input and output markets will always equate the price they are willing to pay for a factor of production to the factor's marginal productivity. In other words, the cost of obtaining one extra unit of input (marginal cost) exactly equals the additional revenue (marginal revenue) that the extra unit of input produces. If the firm achieves this objective it minimizes the cost of producing a given output and thereby maximizes profits. Assuming a given level of output the firm will choose the cost minimizing bundle of inputs by equilibrating the MRTS (or MPL/MPK) to the factor price ratio of inputs (w/r). This axiom is proved with a lagrangian multiplier technique in Appendix B.

If firms in the Alberta sawmill industry can be considered to be cost minimizers then MTRS in Equation 4.24 can be replaced by the factor price ratio of labour to capital (i.e., w/r). The relationship now becomes:

Eq. 4.25 $$\log \frac{K}{L} = \sigma \log \frac{\delta}{1-\delta} + \sigma \log (w/r)$$

Equation 4.25 is the factor proportions equation and the parameters δ and σ can be derived with linear estimation techniques according to:

Eq. 4.26 $$\log K/L = \beta o + \beta_1 \cdot \log (w/r)$$

where $\beta_1 = \sigma$

and $\beta_0 = \beta_1 \cdot \log \frac{\delta}{1-\delta}$

so $$\delta = \frac{\text{antilog } (\beta_0/\beta_1)}{1 + \text{antilog } (\beta_0/\beta_1)}$$

The remaining two parameters (τ, γ) are now derivable by substituting the parameters already estimated (ρ, δ) into the original explicit functional form (Eq. 4.12). In log-linear form equation 4.12 is transformed to:

Eq. 4.27 $$\log Y = \beta_0 + \beta_1 \log (zi)$$

where $B_0 = \gamma$

$B_1 = \tau$

$zi = [\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho}$

Now with equations 4.26 and 4.27 and with values of the appropriate endogenous and exogenous variables, measures of the parameters underlying aggregate production in the Alberta sawmill industry can be estimated.

There still exists, however, a potential aggregation problem in that the sawmill industry in Alberta may not be truthfully represented by one aggregate production relationship but rather by a range of unique production frontiers. For example, shifts in the function could be the result of differences in technology across firms of different size (as was suggested with the Solow model). Or, alternatively, functions may have a unique character across different vintages of capital equipment. The potential influence of these two qualitative influences necessitates a requirement for an approach that accounts for their possible exogenous influences. Two methods exist. The production function can be enhanced by incorporating trend variables into the specification or alternatively by employing a dummy variable specification.

p

C. Procedures

Technical Efficiency in the Alberta Sawmill Industry Cross-Sectionally

The procedures for estimating the index of neutral technical variegation are straightforward and should be evident from Part 1 of the Methodology section.

Data requirements for the model are limited to output (Y), labour (L), capital (K) and capital's share of production (T). Input and output categories are described in detail as follows:

1. Y = Value-added of sawmill operation.
2. L = total man-months in milling operations.
3. K = total new replacement value of all sawmill equipment and infrastructure.
4. T = capital's share of output.

The 83 sawmills, comprising the sample of the Alberta sawmill industry, are segregated into 17 scale classes by first ranking the firms according to increasing output (Y) and placing 5 observations into each scale class. The smallest 5 firms constitute the initial class, the next five firms make up the second class and so on. The last two scale classes each contain 4 firms. The rationale for aggregating mills into classes, as opposed to considering each observation as an independent measurement, is that should errors in measurement have occurred in acquiring the raw data, then possible biases would be more appropriately spread over a larger number of mills. The intent is to minimize possible giant swings in the index caused by inappropriate quantification of inputs or output for individual sawmills.

Simple algebraic techniques are used to derive the index of technical efficiency using Equations 4.8 and 4.9. Equations 4.10 and 4.11 investigate the shape of the aggregate production function, with and without the effects of technical efficiency removed. The aggregate functions are estimated with a simple linear regression model.

There is considerable difficulty in attempting to reach any conclusions as to the statistical properties of the estimated index of technical efficiency. The technique for estimating the shift variable ($\Delta A(s)/A(s)$) is non-stochastic and therefore the assumption must be made that errors in measuring $\Delta A(s)/A(s)$ are non-existent. In fact, if errors in measurement were to occur they would enter the relationship (Eq. 4.5) multiplicatively and would therefore be fully absorbed by the estimated A(s) (Solow, 1957).

Capital Intensity, Returns to Scale, Elasticity of Substitution and Further Aspects of Production Technology

Single-equation estimation techniques are used to derive estimates of the parameters in the C.E.S. model. Specifically the econometric technique employed is Ordinary Least Squares (O.L.S.).

The absolutely necessary data requirements for the model include output (Y), capital (K), labour (L), labour wage rate (w), and capital rental rate (r). Potentially necessary data inputs include a vintage trend variable, a size trend variable, dummy variables to account for size, dummy variables to account for vintage.

With regard to the absolutely necessary information requirements, Y, K and L have been defined earlier and the same descriptions apply for the model employed here. The additional requirements are defined as:

w = amount paid per man-month employed

r = amount paid per dollar of replacement capital employed.

With regard to the potentially necessary data requirements, they are described after their inclusion has been deemed necessary.

Equations 4.26 and 4.27 provide the mathematical relationships necessary for describing production in the sawmill industry. The equations are reiterated as follows:

$$\log \frac{K}{L} = B_0 + B_1 \log (w/r)$$

$$\log Y = B_0 + B_1 \log [zi]$$

The two expressions stated above are exact mathematical formulations and would be deterministic if we can assume an absence of errors in the specific form of the relationship or in the measured values of the variables. There are a number of reasons why one would not expect the relationships to be deterministic.. The following possibilities dictate the necessity of entering a disturbance term in the relationships (Koutsoyiannis, 1977). They include

1. Omission of variables from the function
2. Irrational behavior of firms
3. Imperfect specification of the mathematical form of the model
4. Errors caused by aggregation
5. Errors in measuring the dependent variables.

The disturbance term enters the original function multiplicatively and the stochastic form of the logarithmic relationships appear as (where the disturbance is additive):

Eq. 4.28 $$\log \frac{K}{L} = B_0 + B_1 (w/r) + \mu$$

Eq. 4.29 $$\log Y = B_0 + B_1 \log [\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho} + \mu$$

Using the above expressions, statistical estimates of the

aggregate parameters in the Alberta Sawmill industry are obtainable. A priori speculation, however, indicates that the relationships between input and output are not appropriately represented by the singular functional relationships shown in 4.28 and 4.29. A first order test to validate the existence of one unique function for all firms in the sample is the "Chow Test" for the equality of regression coefficients obtained from different sample sets (Chow, 1960). If the "Chow Test", after dividing the sample in two, indicates that the two estimated functions are significantly different, then the results of functions 4.28 and 4.29 must be rejected.

The procedure for the "Chow Test" is to form an F statistic based on the sum of squares of the residuals ($\Sigma\mu^2$) of three regressions. One value represents the $\Sigma\mu^2$ for the entire sample and the remaining two $\Sigma\mu^2$ are derived after dividing the sample according to the predefined qualitative criteria. The formula for the F statistic is:

Eq. 4.30

$$F^* = \frac{[\Sigma\mu^2 - (\Sigma\mu^2{}_1 + \Sigma\mu^2{}_2)]/k}{(\Sigma\mu_1{}^2 + \Sigma\mu_2{}^2)/(n_1 + n_2 - 2k)}$$

where μ = residual for pooled sample, μ_1 = residual for samples in first part, μ_2 = residual for second part, K = number of exogenous variables, n_1 = number samples in part 1, and n_2 = number of samples in part 2.

If the derived F statistic is greater than the hypothetical value then the regression coefficients are significantly different and the qualitative criteria must be accepted as being influential.

Two separate criteria can be used to divide the observations. If the suspicion is that the production function will vary with size of the operation, the sample would be divided between the largest and smallest firms. Significantly different regression coefficients (as proved by the

test) justifies the introduction of either size trend vvariables or dummy ariables sensitive to size into Equations 4.28 and 4.29.

Alternatively, variations in the production function could result from the vintage of capital employed in the conversion processes. Significantly different functions after dividing the sample according to age, justifies the introduction of either vintage trend variables or vintage dummy variables into 4.28 and 4.29.

For the trend variable specification, four separate sets of equations are possible. The choice of a particular set of equations will depend on the inclusion or exclusion of the two trend variables. The four potential models include:

Model 1 a) $\log K/L = B_0 + B_1 \log (w/r) + \mu$

b) $\log Y = B_0 + B_1 \log [\delta K^{-\rho} + (1 - \delta) L^{-\rho}]^{-1/\rho} + \mu$

Model 2 a) $\log K/L = B_0 + B_1 \log (w/r) + B_2(S) + \mu$

b) $\log Y = B_0 + B_1 \log [\delta K^{-\rho} + (1 - \delta) L^{-\rho}]^{-1/\rho} + B_2(S) + \mu$

Model 3 a) $\log K/L = B_0 + B_1 \log (w/r) + B_2(V) + \mu$

b) $\log Y = B_0 + B_1 \log [\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-1/\rho} + B_2(V) + \mu$

Model 4 a) $\log K/L = B_0 + B_1 \log (w/r) + B_2(S) + B_3(V) + \mu$

b) $\log Y = B_0 + B_1 \log [\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-1/\rho} + B_2(S) + B_3(V)$

$+ \mu$

where S = scale trend variable

V = vintage trend variable

In order that the exact specifications required for estimation in Chapter VI can be described here, it is necessary to jump ahead and observe the empirical results of the "Chow Test". The "Chow Test" results (page 110) show that the production function is insensitive to differences in vintages of capital and sensitive to differences in scale

of production. Thus size is influential and vintage is not. Model 2 is therefore the proper trend variable specification.

What is the appropriate model specification with dummy variables? Arbitrarily a separation of the firms into four size classes has been deemed desirable. Thus, three dummy variables are required (three rather than four variables must be used to avoid the "dummy variable trap").

The model specification of the factor proportions equation with dummy variables is represented by:

Eq. 4.31 $\log K/L = B_0 + B_1 \cdot \log (w/r) + B_2 \cdot DV_1 + B_3 \cdot DV_2 + B_4 \cdot DV_3 + B_5 (DV_1 \cdot \log w/r) + B_6 (DV_2 \cdot \log w/r) + B_7 (DV_3 \cdot \log w/r) + \mu$

where $B_1 = A_1 = \sigma$ = elasticity of substitution (smallest firms)

$B_1 + B_7 = A_2 = \sigma$ = elasticity of substitution (small firms)

$B_1 + B_6 = A_3 = \sigma$ = elasticity of substitution (medium firms)

$B_1 + B_5 = A_4 = \sigma$ = elasticity of substitution (large firms)

and $B_0 = C_1$ = capital intensity factor (smallest)

$B_0 + B_4 = C_2$ = capital intensity factor (small)

$B_0 + B_3 = C_3$ = capital intensity factor (medium)

$B_0 + B_2 = C_4$ = capital intensity factor (large)

Therefore: $\dfrac{\text{antilog } (C_1/A_1)}{1 + \text{antilog } (C_1/A_1)} = \delta$ = capital intensity (smallest)

$\dfrac{\text{antilog } (C_2/A_2)}{1 + \text{antilog } (C_2/A_2)} = \delta$ = capital intensity (small)

$\dfrac{\text{antilog } (C_3/A_3)}{1 + \text{antilog } (C_3/A_3)} = \delta$ = capital intensity (medium)

$\dfrac{\text{antilog } (C_4/A_4)}{1 + \text{antilog } (C_4/A_4)} = \delta$ = capital intensity (large)

The appropriate specification of the log Y equation with dummy variables is represented by:

Eq. 4.32 $\log Y = B_0 + B_1 \log (Z) + B_2 . DV_1 + B_3 . DV_2 + B_4 . DV_3 + B_5 (DV_1$

$\log(Z)) + B_6(DV_2.\log(Z)) + B_7(DV_3.\log(Z)) + \mu$

where $Z = [\delta K^{-\rho} + (1 - \delta)L^{-\rho}]^{-1/\rho}$

$B_0 = \gamma$ = technical efficiency (smallest)

$B_0 + B_4 = \gamma$ = technical efficiency (small)

$B_0 + B_3 = \gamma$ = technical efficiency (medium)

$B_0 + B_2 = \gamma$ = technical efficiency (large)

$B_1 = \tau$ = returns to scale (smallest)

$B_1 + B_7 = \tau$ = returns to scale (small)

$B_1 + B_6 = \tau$ = returns to scale (medium)

$B_1 + B_5 = \tau$ = returns to scale (large)

Before the nature and the statistical properties of the estimates derived in the above models can be characterized, the underlying assumptions of the linear regression model are described. The twelve assumptions described are categorized into two groups:

a) assumptions relating to the specific form of the model and to the nature of the model's variables,

b) stochastic assumptions.

Assumptions in the first group describe desirable conditions for the nature of the relationships between the explanatory variables themselves. Stochastic assumptions are made in reference to the distribution and desirable statistical properties of the disturbance term (μ) and also to the relationships between μ and the explanatory variables. The model assumptions include:

1. The model is linear in its parameters.
2. The model is correctly specified. Therefore, all important explanatory variables are represented explicitly in the model and there is no specification error in determining the relevant

explanatory variables. In addition, for the model to be correctly specified, the mathematical form (i.e., linear, non-linear, simultaneous, single equation, Cobb-Douglas, C.E.S.) must be correct.

3. The assumption that the structure is properly identified ensures that the final structure of the relationships being measured is unique to the relationship expected when the model is specified. A structure is identified when there is only one structure that is admissible. The independent variables specified in the model are related to a unique dependent variable and therefore the coefficients, determined by the model and represented in the structure, properly describe the anticipated relationship.
4. The assumption is made that the variables are correctly aggregated. The use of single categories for factors of production and for output implies that each category is homogeneous. In fact, this may not be the case especially for inputs like capital which comprises a wide array of types of machinery and equipment. Wallis identifies two levels of aggregation problems; at the firm level and at the industry level (Wallis, 1973). Wallis suggests (1973, p. 25)

> "These can be combined into single aggregate variables provided that the marginal rate of substitution between any two kinds of one factor is independent of any variety of the other factor, and these aggregate variables can be treated as if they were actual individual inputs provided that they are linear homogeneous functions of the different varieties."

In cases where Wallis's criteria for an aggregate variable does not hold, it is necessary to either separate broad input categories into more specific groups or to adopt an index approach for compiling aggregate variables.

5. The assumption is made that the exogenous variables are not strongly correlated with each other (or the independent variables do not exhibit a high degree of multicolliniarity).

The stochastic assumptions include:

6. Ui is a random variable which can be positive, negative or zero.
7. The mean of U is zero. Therefore the structural formula describes the relationship on average.

$$Yi = bo + bi\ Xi$$

shows the relationship between X and Y on average.

8. The variance of Ui around its mean is constant at all values of X (on Fig. 4.1 A->B = C->D).
9. The variable Ui is normally distributed (a bell shaped distribution of U's around its zero mean exists for each value of Xi (Fig. 4.1).
10. The disturbance term in sequential observations of the exogenous variables are independent and uncorrelated (cov (Ui Uj) = 0) which ensures no auto correlation.
11. The disturbance terms are uncorrelated (independent) with the explanatory variables (Cov (Xi Ui) = 0).
12. The independent variables are measured without error. The disturbance term assimilates omitted variables and measurement errors in the dependent variable, however, it is assumed that the regressors are errorless.

The value and importance of the previously suggested assumptions is reflected in the desirable properties of the estimates derived in the

D.

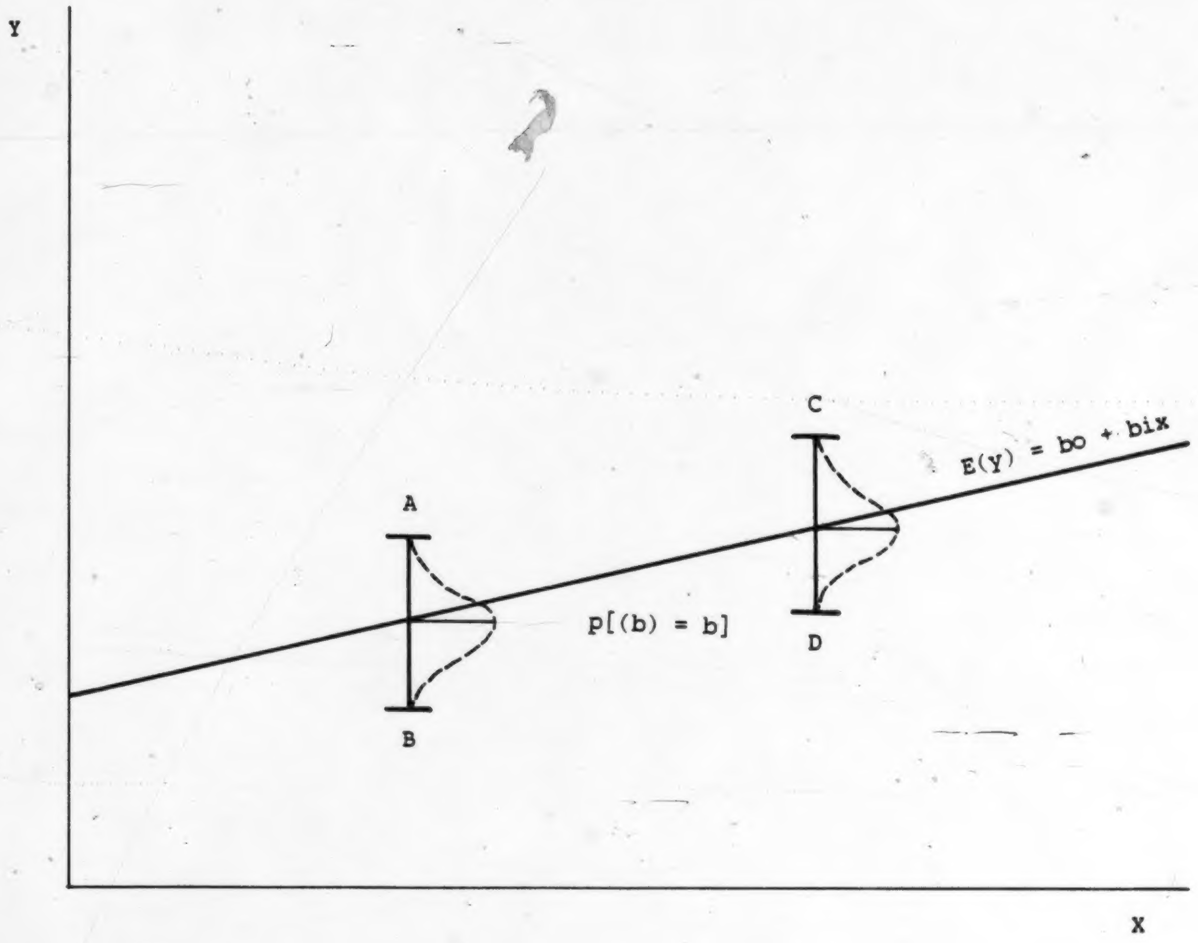

FIGURE 4.1. The Variance of Ui Around Its Mean

models. If each of the suggested assumptions is true, then the derived estimates accept the properties of being (1) best among the class of all efficient estimators, (2) linear, (3) unbiased. The acronym for these desirable properties is B.L.U.E.

The four parameters of interest in the context of the CES production function are: the elasticity of factor substitution (σ), technical efficiency (γ), returns to scale (τ), and capital intensity (δ). The coefficients provided in the equations of the basic model (Eq.'s 4.25 and 4.27) are used to estimate the parameters. First the estimation and statistical properties of the parameters in Equation 4.25 are investigated.

The two coefficients derived in Eq. 4.25 are used to estimate the parameters σ and γ as follows:

$$B_1 = \sigma$$

$$B_0 = B_1 \log [\delta /(1 - \delta)]$$

Eq. 4.33 $$\frac{\text{antilog } (B_0/B_1)}{1 + \text{antilog } (B_0/B_1)} = \delta$$

B_1 is the best, linear, unbiased estimate of σ and the variance of σ is equal to the variance of B_1. Also B_0 is the BLUE of $[\sigma \cdot \log \frac{\delta}{1-\delta}]$. It is not true, however, that;

$$\text{antilog } (B_0/B_1)/1 + \text{antilog } (B_0/B_1)$$

yields a BLUE of δ. Crozier (1980) suggests that with small samples, an upward bias is indicated in the estimate of δ. However, as sample size increases, the bias in the estimate tends toward zero, and therefore Eq. 4.33 provides only a consistent estimate of δ. Wallis (1973, p.59) qualifies further with

> "For the regression equation to produce consistent estimates of ρ (or σ) and δ, we require the relative price ratio to be

independent of the disturbance term, that is, relative prices to be given to the individual firm."

The disturbance term derived in Equation 4.28 is referred to as the commercial efficiency term.[32] It relates the extent to which individual firms are acting in a cost minimization behaviour by utilizing optimal factor proportions with given input prices (MRTS = w/r for given firms).

Having identified Eq. 4.33 as providing a consistent estimate of δ, what is the variance of δ? Kmenta (1971) and Klein (1953) provide a formula for approximating the large sample variance of an estimator, which is a function of any number of other estimators.

Eq. 4.34 $$\mathrm{Var}\ (\alpha) \simeq \Sigma \frac{\partial f}{\partial B} \cdot \mathrm{Var}\ (B\ ) + 2\Sigma \frac{\partial f}{\partial B} \frac{\partial f}{\partial B} \cdot \mathrm{Cov}\ (B\ , B\ )$$

(j, k = 1, 2, 3 ... k; j < k)

Crozier (1980) shows the variance of $\frac{\delta}{1-\delta}$ using Eq. 4.34 to be:

$$\mathrm{Var}\ \frac{\delta}{1-\delta} = \mathrm{antilog}\ [1/B_1\ \mathrm{Var}\ (B_0) + (-B_0/B_1{}^2) \cdot \mathrm{Var}\ (B_1) +$$
$$2[(1/B_1) \cdot \mathrm{Cov}\ (B_0, B_1) + -B_0/B_1{}^2 \cdot \mathrm{Cov}(B_0, B_1)]$$

To this point the statistical properties of the parameters σ and δ have been considered in detail. With respect to the remaining two parameters, Wallis (1973) states that, although not BLUE, B_0 and B_1 provide consistent estimates of technical efficiency (γ) and returns to scale (τ).

γ and τ are consistent estimators as long as:

1. technical efficiency (γ) is independent of the disturbance term (commercial efficiency) in the factor proportions equation (Eq.

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[32] Commercial efficiency is distinguished from technical efficiency. Technical efficiency is definable in terms of observable differences in the productivity of factors of production. Commercial efficiency is the over or under achievement of profit maximizing factor proportions.

4.28).

2. K and L are independent of the disturbance term in the log Y regression (Eq. 4.29).

From equation 4.27

$\gamma = B_0$

$\tau = B_1$

The variances of γ and τ are:

$\mathrm{Var}\ (\gamma) = \mathrm{Var}\ (B_0)$

$\mathrm{Var}\ (\tau) = \mathrm{Var}\ (B_1)$

D. Summary

This chapter has outlined the methodologies and procedures required for evaluating production relationships in the Alberta sawmill industry. Two separate economic models were developed from two explicit functional forms (Cobb-Douglas and Constant Elasticity of Substitution). Also the estimation procedures are described. In addition the basic data requirements of the models are identified. The necessary measure's include; output, capital, labour, capital wage, labour wage, capitals share of output, size trend variable, and three dummy variables sensitive to size. Detailed descriptions along with methods of deriving these variables are provided in the following chapter.

V. THE DATA

A. The Data Requirements Described and Derived

In Chapter IV the primary data requirements for the two basic models were listed. They include measures of output, capital, labour, capital wage, labour wage, capital's share of output, a size trend variable, and dummy variables sensitive to size. The remaining portion of this section defines each data category and describes the calculations required for quantification.

Output (Y): Output is the product of the firm's endeavours. Output can be measured in physical units or in pecuniary magnitudes. For the purpose of this analysis output is defined as the value-added of the sawmill's operations. Equation 5.1 shows how measures of output were derived.

Eq. 5.1

$$Y = \text{Value Added of Sawmill Activities} = \text{Total Sales F.O.B. Mill} - \text{Total Cost of Roundwood Inputs} - \text{Total Cost of Energy}$$

Capital (K): Lipsey, et al. (1973, p. 334) provide an appropriate definition of capital with

> "capital is a man-made factor of production. The supply of capital in a country (or a firm) consists of the stock of existing machines, plant, equipment, etc. This capital is used up in the course of production and the stock is thus diminished by the amount that wears out each year."

For the purpose of this study, capital is the stock of real, physical, man-made goods which are used in the conversion of logs to lumber. Examples of basic capital goods used for lumber production include buildings, sawmill and planing equipment, chipping and drying implements

and mobile yard machinery.

Although conceptually capital is straightforward, its measurement has traditionally proven to be somewhat more complex. As with output, capital may be represented in physical terms or in pecuniary magnitudes. If money units are chosen to be representative then either replacement value or book value can be used to describe the stock of capital.

Because of the array of forms of capital equipment used in sawmilling, physical units are precluded as a measure of capital stock. Alternatively, book value was given initial consideration. However, since many of the mills were completely depreciated (implying a capital stock of zero) book value was rejected as an absolute measure of capital input. The most appropriate measure of the firm's capital stock, and the one adopted here, is replacement value. Replacement value is the cost of replacing the firm's entire infrastructure with new buildings, machinery, and equipment of as similar form as possible. The difficulties with this measure arise from the fact that for some types of equipment (particularly of older vintages) a similar form of new equipment may not be available for comparison. This inherent difficulty, however, is of less significance than biases associated with other measures of the capital stock.

One precaution is necessary for the final measurement of the capital stock to be correct. The value of the firm's capital stock should reflect capital which is actually used rather than capital in place. Thus replacement value must be adjusted to reflect only capital which is utilized (Varian 1978). For this analysis replacement value was adjusted for the number of non-operating months according to the following relation

Eq. 5.2

$$K = \begin{array}{l}\text{Adjusted}\\ \text{Replacement}\\ \text{Value of}\\ \text{Capital Stock}\\ \text{(Mill Only)}\end{array} = \frac{\text{Number of Months Operating Per Year}}{12} \cdot \begin{array}{l}\text{Total}\\ \text{Replacement}\\ \text{Value}\end{array}$$

Labour (L): The factor of production, labour, is defined for this study by the following relationship:

Eq. 5.3 $\quad L = \text{Number of Man-months In Sawmill Operations}$

Capital Rent (r): The wage paid to capital is defined as the return to its use in production. Capital rent therefore reflects the amount paid per unit of capital used in the production process. The total amount paid to capital in a particular production period includes depreciation, expenditures required for maintaining the capital stock and profit. The latter category measures the opportunity cost of capital and risk. Since capital rent is on a per unit basis, the total amount paid to capital per period must be divided by the number of units of capital employed. For this study the capital stock is measured in terms of replacement value and therefore one unit of capital is equal to one dollar. Equation 5.4 describes how capital rent was determined for this study;

Eq. 5.4

$$r = \begin{array}{l}\text{amount paid}\\ \text{per \$ of}\\ \text{capital stock}\end{array} = \frac{\begin{array}{l}\text{Total annual}\\ \text{depreciation}\end{array} + \begin{array}{l}\text{Total annual}\\ \text{maintenance and}\\ \text{repair expenditures}\end{array}}{\text{Total replacement value of the capital stock}}$$

For those mills that did not indicate annual depreciation directly, a value was estimated according to;

Eq. 5.5

$$\begin{array}{l}\text{total}\\ \text{annual}\\ \text{depreciation}\end{array} = \frac{\begin{array}{c}\text{total replacement}\\ \text{value}\end{array} - \begin{array}{c}\text{total book}\\ \text{value}\end{array}}{\text{average age of equipment}}$$

Conspicuous by its absence from the capital rent value provided in Eq. 5.4 is a measure of profit. The potential realization of profits is

the primary justification for investing capital and the abeyance of a measure of profit in r constitutes measurement error. There was, however, no practical means of deriving profit levels for individual firms and therefore the capital rental measure is net of profit returns. One possibility for accounting for a partial measure of profit may be to universally increase r for all firms by an amount equivalent to the average corporate bond rate in 1979. This approach was rejected however since it assumes that all firms in the sample have exactly the same opportunity cost of capital.

Labour wage (w): Labour wage is defined as the payment per unit of labour services (man-month) provided. The value is calculated according to

Eq. 5.6 $$w = \begin{array}{l}\text{amount paid}\\ \text{per man-month}\\ \text{employed}\end{array} = \frac{\text{total wages and salaries paid}}{\text{total number of man-months employed}}$$

Capital's Share of Output (T): The value of T is equivalent to the elasticity of production for a factor input (in this case capital) discussed in Chapter III. The position of T in the Cobb-Douglas production function is shown in Chapter IV (Eq. 4.4). Capital's share of output is defined as the proportion of total output paid to capital. Equation 5.7 shows the empirical derivation of this relationship.

Eq. 5.7 $$T = \begin{array}{l}\text{Capital's Share}\\ \text{of Output}\end{array} = \frac{r \cdot K}{Y}$$

where r, K and Y are as previously defined.

Size Trend Variable (S): The purpose of the scale trend variable is to enable a delineation of the sample firms according to size of operations. Seventeen size classes were arbitrarily designated. The 83 sawmills in the sample were assigned to size classes by first ranking the firms according to increasing Y. The smallest five firms were then

assigned to the first size class which was identified by the variable number = 1. The next five mills are identified by a size trend variable number = 2. There are five mills in each of the first 15 size groups. Size classes 16 and 17 have four firms each. Table 5.1 shows the number of firms and the mean average output (Y) and mean physical production for each size class.

Dummy Variables Sensitive to Size (DV_1, DV_2, DV_3): As indicated in Chapter IV, dummy variables are required in the C.E.S specification to accommodate the qualitative influence of size on the production parameters. Dummy variable one (DV_1) delineates the largest 21 firms in the sample. Dummy variables two and three describe firms 22-42 and 43-63 respectively. The smallest 20 firms are not accounted for by any of the three variables.

B. Data Collection Procedures

The original source of the data used in this study is the primary wood using industries survey conducted by staff of the Northern Forest Research Centre (NoFRC)[33] associated with the project *Resource Opportunities and Policy Guidelines* (NOR-3).

Although the data used in this analysis is primary, it was not collected with the intention of use in an analytical study. The data was solely intended for providing descriptive information on Alberta's primary wood-using industries (which includes sawmills). The discovery of the suitability of the information for production function analysis was serindipitious but after the fact.

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[33] The NoFRC is a regional research facility of the Canadian Forestry Service (Department of the Environment, Government of Canada). The centre is responsible for addressing research and technology transfer needs in the three prairie provinces and in the Northwest Territories.

TABLE 5.1
Number of Firms, Mean Output, Mean Physical Production By Scale Class

Scale Class (Trend Variable Number)	Number of Firms	Mean Output $/year/firm	Mean Physical Production fbm/year/firm
1	5	5,763	61,400
2	5	9,777	132,800
3	5	13,033	188,600
4	5	20,222	217,600
5	5	23,996	216,200
6	5	30,588	332,200
7	5	36,121	360,000
8	5	43,364	535,000
9	5	53,027	497,140
10	5	69,547	753,000
11	5	92,834	788,530
12	5	15,533	1,658,000
13	5	377,210	1,698,259
14	5	588,938	7,080,950
15	5	1,378,645	14,458,568
16	4	2,096,861	14,480,775
17	4	5,548,970	39,336,248

The primary wood-using industry survey was conducted by three members of the NoFRC from May to November, 1979. The approach was to individually interview each firm within the primary wood-using industries. Although a complete census of all firms in the province was achieved, only a proportion of the operations were subjected to a long detailed questionnaire. The remaining firms were asked to respond to a much less specific questionnaire. The substance of the less detailed questionnaire lacked the detail necessary for this analysis and was eliminated.

With respect to the sawmill industry, 133 of the 394 mills in the industry were interviewed with the detailed questionnaire. The degree and accuracy of response to this detailed questionnaire was highly variable. Some firms provided a minimal amount of information of dubious

accuracy and other firms responded in detail and with great care to the complete set of questions.

From the group of detailed questionnaires a subset of questionnaires was accumulated. The criteria for inclusion of individual questionnaires in the subset was an information set complete enough to satisfy the data requirements described in the first part of this chapter. Questionnaires with obvious inconsistencies were culled from the subset. The final result was a complete set of information for 83 firms in the Alberta sawmill industry. Table 5.2 shows the distribution of these 83 firms by size category along with the distribution of those firms subjected to the original detailed questionnaires and the population of Alberta sawmills in 1979.

C. Statistical Representativeness of the Sample

The 83 firms used in this analysis do not represent a statistically derived, representative sample of the population of Alberta sawmills. Thus there is a potential for bias. To test for representativeness, the sample and population means of three yardstick variables will be tested for homogeneity. The variables are production, average haul (one-way) woods to mill, and total man-months of employment.[34] Because of the small number of sample mills in the smallest mill class, only the 183 mills in the first three size classes are considered as the population of Alberta sawmills. Thus all conclusions made in this thesis are relevant only to firms producing greater than 100 M fbm per year.

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[34] All the specific variables used in the analyses (Capital, Value Added, Capital Wage, Labour Wage) could not be tested because they could not be obtained for the population set.

TABLE 5.2
Distribution of Alberta Sawmills by Size Group

Size Class Annual Output - M fbm	Total Number of Alberta Sawmills (No. of firms)	Sawmills Subjected to Detailed Questionnaire (No. of firms)	Sawmill Sample For This Analysis (No. of Firms)
greater than 5,000	26 (7%)	23	18 (22%)
1,000 - 5,000	29 (8%)	22	12 (14%)
100 - 1000	128 (32%)	69	47 (57%)
less than 100	211 (53%)	19	6 (7%)
Total	394	133	83

The statistical test[35] used to evaluate the similarity of the population and sample means is the "t"-test. Since this test assumes that the sample and population variances are identical an initial test to evaluate the presence of homogeneous variance is required. The statistical test used for evaluating the homogeneity of the sample and population variance is the "F"-test. The F statistic is formed by the ratio of the population and sample variances. If the variances are homogeneous, the F statistic will equal one. Details regarding the format of testing equations are provided in Appendix C.

Table 5.3 shows the derived t* and F* statistics based on the sample and population means and variances. The critical values with the appropriate degrees of freedom are;

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[35] The statistical procedures used here are from Alder, H. L. and E. B. Roessler, 1972, Introduction to Probability and Statistics, 5th Edition, W. H. Freeman and Company.

TABLE 5.3
Derived t and F Values for Testing The Homogeneity of Sample and Population Variances and Means

Variable	t*	F*
Production	.074	1.39
Average Haul	1.48	1.038
Employment	.25	1.220

$F_{95(182\ 82)} \simeq 1.39$

$t_{95}\ (_{264}) = 1.97$

Since the derived measures of F* for each of the three variables are less than or equal to the critical value, the assumption of homogeneous variances is acceptable. Also, since the derived measures of t* for each of the three variables are less than the critical value, the assumption of homogeneous means is also acceptable. Thus the conclusion can be made that the sample of firms used in this analysis accurately represents all Alberta sawmills producing greater than 100 M fbm per year.

D. Summary

This chapter has focussed on the information requirements of the models and the statistical characteristics of the sample. In Section A, the data categories were described in detail. Section B outlined the collection procedures and section C provides proof that the sample is representative of the population. The following chapter incorporates the data described in this chapter, with the models developed in chapter IV to provide empirical measures of the production relationships describing Alberta sawmill production.

VI. RESULTS AND DISCUSSION

A. Introduction

In this chapter the empirical estimates of the parameters of the two basic models developed in chapter IV are presented and analyzed. In addition the aggregate Cobb-Douglas production function is estimated and the outcomes of the Chow test are provided.

B. Aggregate Cobb-Douglas Production Function

Equations 1.2 and 4.4 represent the linear-homogeneous Cobb-Douglas production function in explicit form. A more generalized version of this function is

Eq. 6.1 $$Y = A \cdot K^{\alpha} \cdot L^{\beta}$$

where α = production elasticity of capital

β = production elasticity of labour.

The purpose of deriving the Cobb-Douglas production function is to obtain an initial understanding of production relationships in the Alberta sawmill industry. The information is important for judging the validity of assuming constant returns to scale in the Solow model.

The estimated form of the aggregate, unrestricted (in terms of linear homogeneity) Cobb-Douglas production function is:

Eq. 6.2 $$\log Y = 5.84 + \underset{(0.6)}{.78 \log L} + \underset{(.08)^{36}}{.25 \log K} \quad (R^2 = .93)$$

F statistic = 563.78

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[36] These values are standard errors and they indicate with a t test that the coefficients are significantly different from 0 at the 99% confidence level.

The standardized[37] Cobb-Douglas coefficients (Beta weights) for the Alberta sawmill industry in 1978-79 are .285 and .699 for capital and labour respectively. The magnitudes of the individual production elasticities provide some interesting results. Each production elasticity (or factor share) represents the rate of change of output which results from a proportionate change in a factor input (holding all other factors of production constant). The production elasticity of capital is .285 therefore a 10% increase in the capital stock in Alberta will result in a 2.85% increase in output. A 10% increase in labour would cause output to increase 6.99%.[38] A 10% increase in both labour and capital would cause output to increase 9.84% (2.85+6.99). An important point is that the production elasticities measure proportionate output responses with small changes in individual factors. For example, although output might increase 2.85% with a 10% increase in capital, even greater increases in capital will not have the same proportionate output response. This is due to the law of diminishing marginal productivity. If more capital is used in the production process relative to labour, capitals marginal product declines and therefore the production elasticity of capital will decline. This facet is addressed in the following paragraph where another coefficient is introduced which monitors the effects on marginal productivity of a given change in the quantity of the factor employed.

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[37] The coefficients are standardized so they have unit variance (i.e., standard deviation of X and Y = 1). The equation for standardizing is:
Eq. 6.3 Standardized B = (Actual B)·
[(standard deviation X)/(standard deviation Y)].
This procedure is necessary to compensate for the different measurement units of K and L. The beta weights provide a better measure of the relative effects of each independent variable on the dependent variable.

[38] Layard and Walters (1978, p. 274) show that when $\sigma = 1$, then $dx/x = \alpha\, dK/K + \beta dL/L$.

Gunn and Douglas (1940) indicate two additional parameters which are derived from the individual production elasticities. They term these values the *coefficient of flexibility* (Ø) and the *coefficient of elasticity* (λ). The coefficient of flexibility represents "the relative rate at which the marginal productivity changes with a given proportionate change in the quantity of a factor" (Gunn and Douglas, 1940, p. 400).[39] Numerically this coefficient is equivalent to the production elasticity of the factor minus one. In the Alberta industry Ø is -.715 and -.301 for capital and labour respectively. Therefore a 1% increase in the quantity of capital (with labour constant) results in a .715% decline in the marginal productivity of capital. Alternatively a 1% increase in quantity of labour causes a .301% decline in the marginal productivity of labour.

The coefficient of elasticity is determined by dividing the relevant Ø into 1. Thus λ is -1.399 and -3.322 for capital and labour respectively. The interpretation is that a 1% increase in labour wage[40] would cause a decrease of about 3.3% in the number of workers employed.

Another significant result of the regression for the Alberta sawmill industry is provided by the sum of the production elasticities which is very close to one. This feature implies a linearly homogeneous production function which is synonymous to stating that production technology in the industry exhibits roughly constant returns to scale. Thus a doubling of both factor inputs will very nearly double output. This structural feature of the industry's production technology has not only significant descriptive merit but also has important implications

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[39] See Gunn and Douglas (1940) for mathematical proof.

[40] Assuming that there is perfect competition in factor markets and that wages are determined by the marginal productivity of each factor.

with regard to the assumption of a linear homogeneous production function employed in Solow's model.

The interpretation of the ratio of the production elasticities also provides some provocative results. Since each elasticity represents the amount paid to each factor out of total product, the magnitude of α/β is indicative of the relative factor intensity of the production technology in a particular region. For example the α/β ratio is much higher in the Alberta sawmill industry (.408) than the Philippine sawmill industry (.150) (Laarman, 1982). Thus, assuming that the industry in both regions is appropriately described by the same production function, the Alberta sawmill industry is relatively more capital-intensive than the Philippine industry[41].

C. Capital Deepening and Technological Variegation[42] in the Alberta Sawmill Industry Cross-Sectionally

From Table 6.1 the general trend of increasing output per unit of labour (Y/L) with increasing scale of operation is clearly evident. The Y/L ratio among the smallest scale class is $1,108 and increases by 391% to $4,329 in the largest size class. Concurrent with this productivity

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[41] This fact is entirely consistent with the a priori expectation that in the more developed regions of the globe, the higher unit labour costs cause a substitution of capital for labour to occur resulting in more capital intensive production processes.

[42] The term technological variegation is used in a special sense here in that technological change is normally a concept that reflects changes in production relationships resulting from technological innovation over time. However, in that firms in the sawmill industry utilize diverse technologies with heterogeneous inputs, measurable differences in the production relationships can be attributed to changes in technologies. Thus for the remainder of this thesis technological variegation reflects cross-sectional qualitative changes in the factor inputs. A problem with this approach is that technical efficiency can not be distinguished from technological change. As discussed in the final chapter, this is an empirical question requiring further study.

TABLE 6.1

Indices of Cross-Sectional Technological Efficiency and Components of The Index

Size Class	Net Sales Y $	Capital Stock K $	Labour L person months	Y/L $	K/L $	Capital Share of Output T	A(s)/A(s)	A(s)	Corrected Y/L (Y/L/A(s))
1	28,815	46,750	26	1108	1798	.408	-	1.00	1108
2	48,887	84,600	34	1438	2488	.427	.141	1.141	1260
3	65,165	39,370	46	1417	856	.224	.265	1.443	982
4	101,108	52,250	43	2351	1215	.293	-.565	2.259	1041
5	119,981	143,218	65	1846	2203	.201	-.453	1.236	1494
6	152,938	92,050	96	1593	959	.234	.024	1.206	1321
7	180,607	140,133	103	1753	1360	.229	.003	1.210	1449
8	216,821	174,680	100	2168	1747	.211	.172	1.418	1529
9	265,136	163,410	95	2991	1720	.214	-.383	1.961	1525
10	347,734	127,180	165	2070	757	.135	-.188	1.592	1300
11	464,168	276,020	228	2036	1211	.196	.097	1.437	1417
12	777,665	429,725	266	2924	1615	.185	-.371	1.971	1483
13	1,886,050	2,519,220	502	3757	5018	.164	-.105	1.764	2130
14	2,944,689	2,093,400	1141	2581	1835	.196	-.027	1.395	1850
15	6,893,223	5,512,742	2681	2571	2056	.202	-.027	1.358	1893
16	8,387,445	14,907,000	2687	3121	5548	.272	-.127	1.182	2640
17	22,195,880	29,840,496	5127	4329	5820	.198	.374	1.625	2664
Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10

improvement is a general increase the capital-labour (K/L) ratio from \$1,798 to \$5,820. The results presented in this section show, by way of Solow's model, how much of the productivity improvement is the result of increased total factor productivity and how much is the result of strictly capital deepening in larger sawmills.

Table 6.1 shows that the index of technological variegation increases about 62% (column 8, A(s) for scale class 17 = 1.625) from the smallest firms to the largest firms indicating a significant difference in the production technology employed by small and large mills. Although this overall general increase has occurred the rate of the increases from size class to size class has not been stable. Examination of column 9 shows that technological variegation makes a very significant contribution to productivity improvement in the early stages of the cross-section series and then drops drastically in size class five. The index then continues to fluctuate between 1.2 and 1.9. Although disconcerting, the unstable movement of the index is not entirely unexpected. There is a wide variation in processing methods in the middle classes of the sawmill industry. An undulating index in the presence of these differences is therefore not a surprising phenomenon. Information on the inter-scalar movement of the index is not as important for the purposes of this study as the overall result.

Column 10 of Table 6.1 shows output per unit of labour after netting out the influence of technological variegation. The values in this column represent the growth in productivity resulting from increased capital intensity only. Capital deepening accounts for 48% of the growth in the productivity of labour between small and large mills. The remaining 52% is caused by technological variegation. Figure 6.1

shows these effects graphically. Productivity improvements caused by moving along curve 1 are the result of the employment of increasingly capital intensive production processes. Technological variegation causes a complete shift in the production function to a higher level. Thus at each K/L ratio a greater Y/L can be obtained and curve 1 shifts to curve 2.

The actual shapes of curves 1 and 2 are provided by the estimated versions of Equations 4.10 and 4.11 respectively. The results of these regressions are:

Eq. 6.4 $\log(Y/L) = -.181 + 1.031 \log(K/L)$ $R^2 = .379$

Eq. 6.5 $\log(Y/L/A(s)) = -2.266 + 1.738 \log(K/L)$ $R^2 = .682$

These two equations represent the aggregate production functions for the Alberta sawmill industry both with and without the effects of technologies removed.

The statistical reliability of the Solow model results is impossible to determine. The equation used to derive the index of technological efficiency is non-stochastic and thus the presumption must be made that biases have not occurred. This premise is highly unlikely. A discussion of the results in light of potential sources of bias therefore seems appropriate. Two potential sources of bias include both measurement error and inappropriate assumptions.

There are undoubtedly errors embedded in the measured values used with this model. Because this possibility is discussed briefly in Chapter V, it is not given further attention here. Suffice to say that the data used are the most complete available for representing the full cross-section of Alberta sawmills.

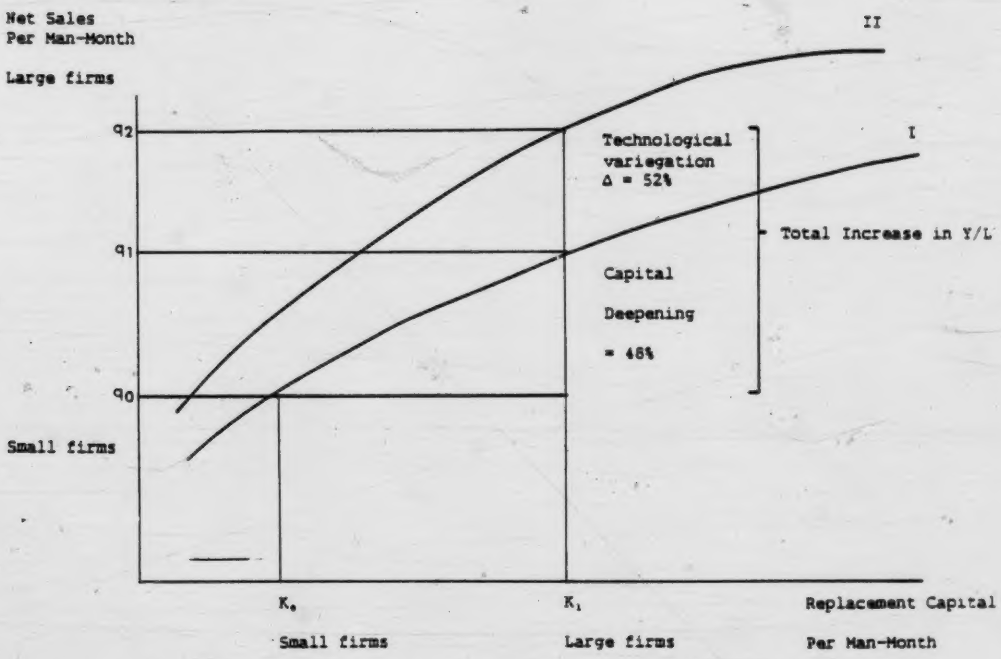

FIGURE 6.1 The Effect of Technological Variegation and Capital Deepening in Alberta Sawmills

Of equal relevance as a potential source of bias is the effect of the basic assumptions embedded in the employment of the Solow model. These assumptions are introduced on page 71.

The first assumption, which states that technological variegation must be "Hicks" neutral, is necessary to ensure that technological differences results in absolute changes in the coefficient (A(s)). If technological variegation is in fact factor augmenting then the marginal rate of technical substitution is affected such that the resulting productivity improvements are not reflected in A(s). The rapidly increasing K/L ratio in larger scale plants suggests the possibility that there has been a technology induced change in MRTS and thus that technological variegation is non-neutral. The trend of increasing K/L ratio can also be explained as a response to changing factor price ratios. Casual observation of labour wage data for increasing plant scales does show an increase in unit labour costs. Thus the increasing K/L ratio can be explained as a direct substitution effect resulting from increases in the price of labour relative to capital. This effect is compatible with the concept of neutral technological change and the assumption is therefore presumed to be valid.

The second assumption states that there is perfect competition in factors of production markets. This assumption ensures that inputs are paid at a rate equal to their marginal products. In the Alberta sawmill industry the assumption of perfect competition is a valid one.[43] No single firm or group of firms in the industry is in a monopsonistic or oligopsonistic position in terms of being able to affect or alter the prices of inputs (Chapter 2). Assuming firms behave rationally and since

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[43] At least in terms of the assumptions that firms are price takers and profit maximizers.

prices of inputs are given, the assumption that factor inputs are paid according to the marginal productivity of each input is acceptable.

Implicit in assuming a linear homogeneous Cobb-Douglas production function are (1) the assumption of a constant return to scale production technology, and (2) a unitary elasticity of substitution between capital and labour. If these assumptions are proved wrong then the functional form used in the model does not properly specify the production relationships. The authenticity of the second assumption is addressed in the fourth section. For now it must accepted as possibly a necessary abstraction from reality. The first assumption however is particularly powerful with respect to its implications on the value of A(s). If constant returns to scale do not exist then Equation 4.5 cannot be created from Equation 4.4 and A(s) cannot be estimated. The results presented in the first section of this chapter show that aggregate production technology in the Alberta industry does exhibit roughly constant returns to scale. Therefore the assumption of a linear homogeneous production function is fundamentally valid.

D. Chow Test for Equality of Regression Coefficients

Solow's model has shown that the Alberta sawmill industry cannot be truly characterized by one homogeneous production frontier. Interscalar differences in technology clearly cause the function to shift. The existence of the unstable function emphasizes an important empirical question: Is it possible that shifts in the function could be caused by qualitative influences other than size? For the purpose of this study the possibility of shifting functions resulting from two separate qualitative criteria are tested. The two criteria are again

size of operation and vintage of equipment.

Table 6.2 shows the results of the five separate regressions required for testing the effects of size and vintage on the production relationships. From the information in Table 6.2 the F-statistic formed to test for the effect of size is:

Eq. 6.6 $$F^* = \frac{[SS\ all - SS\ small + SS\ large)]/2}{(SS\ small + SS\ large)/79} = 6.8215$$

The critical value of F(0.05) ($v_1 = 2$, $v_2 = 79$) is 3.12. Since $F^* > F(0.05)$ then the conclusion is made that there is a significant difference in the regression coefficients and size does cause the production function to shift.

The F statistic formed to test for the effect of vintage of equipment is:

Eq. 6.7 $$F^* = \frac{[SS\ all - (SS\ young + SS\ old)]/2}{(SS\ young + SS\ old)/79} = 1.82204$$

Since $F^* < F(0.05)$ then the conclusion is made that there is not a significant difference in the coefficients and vintage does not cause the production function to shift.

The results of these tests are vital for deciding which endogenous variables will be included in the CES model. The "Chow Test" has shown that size is the only influential variable.

E. The Constant Elasticity of Substitution Production Model

The CES model is the more complex of the two basic models employed in this thesis. The justification for the use of this more sophisticated model is that the model is less restrictive, is more informative, and is statistically more reliable than the Solow model. The CES model is less restrictive because (1) the model is not constrained to linear

TABLE 6.2
Results of Regression Runs for Testing Differences In Coefficients After Separating Sample According to Size and Vintage

Sample Description	Code for Sum of Squares	Regression	R^2	Sum of Squares of Residuals	Degrees of Freedom
Pooled Data	SS all	$\ln K/L = -.74 + .94 \ln w/r$	.26	52.19241	81
Small Firms	SS small	$\ln K/L = -1.51 + 1.01 \ln w/r$	.26	30.69412	40
Large Firms	SS large	$\ln K/L = -.84 + .80 \ln w/r$	.30	13.81219	39
Young Firms	SS young	$\ln K/L = 0.47 + .94 \ln w/r$	.23	25.26629	36
Old Firms	SS old	$\ln K/L = -1.41 + 1.01 \ln w/r$	.32	24.62476	43

homogeneity, and (2) the model does not presume a unitary elasticity of substitution. The model provides more information because (1) the actual nature of returns to scale can be estimated, (2) the true elasticity of substitution can be derived. The model is statistically more reliable because it is stochastic and is therefore subject to a complement of statistical tests which provide a degree of credibility to the results.

The results of the "Chow Test" in the previous section suggest that of the two qualitative variables tested, size had the only significant effect on the regression coefficients. Thus a single homogeneous function does not represent sawmill technology and some specification which allows for size effects must be employed. Two separate specifications were in fact estimated. Equations 6.8 and 6.12 represent the estimated versions of the trend variable specification and equations 6.14 and 6.15 represent the production function with dummy variables.

The empirical results of the factor proportions equation with a size trend variable were:

Eq. 6.8
$$\log (K/L) = \underset{(1.431)^{[**]}}{-.510} + \underset{(.170)}{.861} \log (w/r) + \underset{(.017)}{.062}.(S)$$

$R^2 = .36$

$F = 22.424$

A first order F test to test for the overall significance of the regression coefficients shows that the hypothesis that the overall regression is significant at the 95% confidance level is valid. Thus in spite of the fact that the coefficients of determination is low (.36) the estimated regression has some explanatory power. First order t tests

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[**] All values in brackets are standard errors.

of the significance of the individual coefficients verify the hypothesis that the coefficients B_1 and B_2 are significantly different from zero at the 95% level.

A general observation of the distribution of the residuals does not indicate the presence of heteroscedacticity.

From equation 6.8, the directly estimated elasticity of substitution (σ) is .861. First order tests of this parameter show that it is significantly different from 0 and thus the Leontiff assumption of zero elasticity of substitution is incorrect. The Cobb-Douglas hypothesis of unitary elasticity of substitution could not be rejected since the parameter is not significantly different from one. The estimated value of σ indicates that a 10% change in the ratio of factor prices causes a 8.6% change in factor proportions.

The estimated value of the capital intensity parameter (δ) is .204. From equation 4.34 the variance of $\frac{\delta}{1-\delta}$ is 2.669. A first order t test shows that the capital intensity parameter is not significantly different from zero.

The coefficient associated with the size trend variable provides the opportunity for making some valuable interpretations regarding the nature of technological variegation.

The value indicates that capital intensity increases 6.2% for each subsequent size class. This trend is significant at the 99% confidance level. Stier (1982) shows that the coefficient B_2 can also be used to calculate the difference in the technology change induced rates of factor augmentation across sizes according to the following relationship;

Eq. 6.9 $B_2 = (1-\sigma)(\lambda l - \lambda k)$

where: λl = rate of labour efficiency improvement

λk = rate of capital efficiency improvement.

Equation 6.9 can be transformed to;

Eq. 6.10 $$(\lambda l - \lambda k) = \frac{B_2}{(1-\sigma)} = \frac{.062}{1-.861} = .446$$

The value of $(\lambda l - \lambda k)$ indicates that technological variegation has increased the efficiency of labour at a much greater pace than capital. From Eq. 6.10 the technological variegation bias can be derived according to;

Eq. 6.11 $$\Phi = -\frac{\sigma-1}{\sigma}(\lambda l - \lambda k)$$

$$= -\frac{.861 - 1}{.861} \cdot (.446) = .072$$

Since the value is positive, the technological variegation bias is labour saving.

Equation 6.12 shows the empirical results of the linearized explicit CES function with a size trend variable.

Eq. 6.12 $$\log Y = \underset{(.165)}{6.112} + \underset{(.05)^{45}}{.543} \log (z) + \underset{(.02)^{45}}{.190} (S)$$

$$R^2 = .97$$

$$F^* = 1139.19$$

From equation 6.10 the estimated technical efficiency parameter (γ) is equal to 6.112 and the returns to scale parameter (τ) is equal to .543. The value of the parameter τ indicates declining returns to scale and a 1% increase in both capital and labour would cause only a .543% increase in value-added. The value of the coefficient associated with the size trend variable indicates the existance of Hicks neutral or product augmenting technological variegation occurring at a rate of 19%

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[45] Significantly different from zero at 99% confidence level.

with each increase in size class.

According to the estimations provided in equations 6.8 and 6.12, the aggregate explicit CES production function for the Alberta sawmill industry is;

Eq. 6.13 $Y = 6.112 \left[.204 K^{161} + .796 L^{161} \right]^{3\ 373}$

However, the inferred existance of biased technological variegation implies that a single set of parameters is not relevent for representing sawmill technology.

The dummy variable specification provides the opportunity of quantifying changes in the parameters caused by scale. The estimate of the dummy variable specification of the factor proportions equation is;

Eq. 6.14 $$\log \frac{K}{L} = \underset{(1.315)}{-.751} + \underset{(.155)}{.946} \log \frac{w}{r} + \underset{(.185)}{.717} (DV_1) - \underset{(.186)}{.453} (DV3)$$

$R^2 = .47$

$F^* = 23.081$

The slope adjusting dummy variables were not significant and were therefore not included in the equation. Also the intercept adjusting dummy variable coefficient for medium firms (B_3) was not significant. Significant coefficients were B_1, B_2 and B_4 (see equation 4.31). The coefficient of determination (.47) indicates that the dummy variable specification provides a better model than the trend variable specification (R^2-.36).

The model provided in equation 6.14 suggests an elasticity of substitution of .946 for all firms in the Alberta sawmill industry. This value is significantly different from zero but insignificantly different from 1 at the 90% confidence level. Thus the Cobb-Douglas assumption of unitary elasticity of substitution is acceptable.

An interesting result of equation 6.14 is the size influenced variation in the capital intensity parameter. The value of the parameter for large, medium, small and smallest firms is .491, .311, .219 and .311 respectively. These results suggest that the large mills are more capital intensive than smaller sawmills. The results verify the capital deepening trends identified by the Solow model and the scale trend variable - CES specification. The smallest mill group is an exception. The capital intensity parameter is actually lower in the small group than in the smallest group. A possible explanation of this inconsistency is that statistical artifacts in the capital input values for the smallest mills, may have occurred as a consequence of the requirement for imputting capital value to obsolete equipment. Because of the general existance of newer equipment, this procedure would have less of a biasing influence in the larger mills.

The empirical results of the linearized explicit CES production function in a dummy variable specification are;

Eq. 6.15

$$\log Y = 6.8 + .647 \log(Z) + .232\,(DV_1 \cdot \log(Z))$$

$$(.265)\ (.065)\qquad (.035),$$

$$+ .121\,(DV_2 \cdot \log(Z)) + .067\,(DV_3 \cancel{\cdot \log(Z))}$$

2

$$(.019)\qquad\qquad (.015)$$

$$R^2 = .96$$

$$F^* = 449.804$$

Each of the slope adjusting dummy variable coefficients are significant and none of the intercept adjusting dummy variable coefficients are significant. Statistically significant coefficients include B_0, B_1, B_5, B_6, and B_7 (see equation 4.32).

The technical efficiency parameter is equal to 6.80 and is invariant to differences in scale of operation. Variation of this parameter would reflect Hick's neutral (or product augmenting) technological variegation. The invariance of the parameter to scale is therefore surprising in view of the supportive inferences regarding neutral technological variegation provided in the Solow, and CES trend specification models.

The returns to scale parameter is unique for each of the different production classes. The results indicate that returns to scale are; .88 for large firms, .77 for medium scale firms, .71 for small firms, and .65 for the smallest firms. Two important conclusions can be drawn from these observations. First, overall sawmill production technology is characterized by decreasing returns to scale. Second, production technologies for each of the individual output classes are characterized by lower levels of returns to scale for smaller output levels. For example, a doubling of all inputs in the smallest mills would result in only a 65% increase in output. For the small, medium and large sawmills, a doubling of capital and labour cause a 71%, 77% and 88% increase in output respectively. Shepherd (1979) identifies three specific sources of diseconomies[46] of scale. They include; a fixed factor, bureaucracy, and transportation costs. Each of these influences have some varying effect on returns to scale in Alberta sawmills. The fixed factor constraint is identified by Shepherd as being primarily a limited managerial capability. This facet is particularly influential in small, owner-operated sawmills where the proprietor must provide all the

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[46] Theoretically economies (or diseconomies) of scale and returns to scale are different but related concepts. The influences that cause these affects however are similar enough that a distinction is not warrented for this thesis.

managerial functions for the operations. As the operation expands, management decisions become both greater in number and more complex. Thus the managerial capability of the single owner-operator becomes less informed and more diluted. Operational efficiency is, in turn diminished.

The effect of bureaucracy (the second source) is described by Shephard (1979 pg. 234) as follows: "As size increases, the manager must delegate tasks. Committees, staff, and layers of middle managers arise. Information is passed up, but it is subject to distortion. There is no complete substitute for first hand contact. Orders are passed down, but they too can become ineffective. Therefore bureaucracy adds direct costs - of staff, offices, memo writing, etc. It also reduces the quality of decision making. All of this tends to make average costs higher than otherwise as size increases." This influence is probably more prevailing in the large mills.

The third source of declining returns to scale is unit transportation costs. This effect can be reflected in higher average costs of transporting both inputs and outputs. If a particular sawmill increases output, then that plant will be faced with the dual prospects of having to move further from the plant to both procure roundwood and to market the final product. For plants governed by rising average transportation cost curves, diseconomies of scale are realized.

Intuitively, the first source is probably more important for small firms and the second source for large firms. The third source may be relevant for all sizes of sawmills. The empirical results indicate that the overall effects of the three causes are more prevailing in smaller mills than in large mills.

The dummy variable specification used for estimating the parameters of production provided by the CES production function has shown that four separate sets of parameters are required for appropriately representing Alberta sawmill production technology. A unique set of parameters exists for each of four separate scales of operations. The parameters are shown in table 6.3. Associated with these parameters are four unique explicit CES production functions. These equations are shown as follows;

Eq. 6.16 a) $Y = 6.8\ [\ .491\ K^{-057} + .509\ L^{-057}]^{-15\ 44}$ large mills

b) $Y = 6.8\ [\ .311\ K^{-057} + .689\ L^{-057}]^{-13\ 51}$ medium mills

c) $Y = 6.8\ [\ .219\ K^{-057} + .689\ L^{-057}]^{-12\ 46}$ small mills

d) $Y = 6.8\ [\ .311\ K^{-057} + .689\ L^{-057}]^{-11\ 40}$ smallest mills

This chapter has presented the empirical estimates of the two basic models developed in chapter IV. In addition the aggregate Cobb-Douglas function has been estimated and a Chow test performed to evaluate the qualitative influence of scale and vintage.

The following chapter summarizes the empirical findings presented and discusses the general policy implications of the results. In addition the models are evaluated as to their representativeness of sawmill production technology. Also avenues of further research are briefly discussed.

TABLE 6.3
Parameters of Production By Scale Group in the Alberta Sawmill Industry - 1979

Size Group	Technical Efficiency γ	Capital Intensity δ	Returns to Scale τ	Elasticity of Substitution σ
large mills	6.80	.491	.88	.946
medium mills	6.80	.311	.77	.946
small mills	6.80	.219	.71	.946
smallest mills	6.80	.311	.65	.946

VII. SUMMARY AND CONCLUSIONS

A. Summary of the Empirical Results

In Chapter One, the specific objectives of this thesis were outlined. Briefly, these objectives included:

1. to determine the nature of returns to scale;
2. to isolate the productivity effects of technological variegation and capital deepening;
3. to determine the technological change bias
4. to establish the influence of size and vintage on the production relationships;
5. to determine the substitutability of factors of production;
6. to evaluate the alternative methodologies;
7. to demonstrate the implications of the empirical measures with respect to the effective implementation of policy; and
8. to suggest a methodology for further and more sophisticated analysis of forest industries.

Objectives 6, 7 and 8 form the basis of this final chapter and are addressed in sections B, C, and D respectively. The results satisfying the empirical objectives (1 to 5) are provided in the previous chapter and are summarized as follows.

With regard to objective 1, the production technology for each sawmill size group is characterized by unique values for returns to cale. According to the CES dummy variable specification, returns to scale are .88, .77, .71 and .65 for large, medium, small and smallest firms respectively. Thus returns to scale are declining at all levels of production but are higher for larger mills.

With regard to Objective 2, the index of technological change, according to the Solow model, increases about 62% between the smallest sawmills and the largest mills. Of the total increase in output per unit of labour, 48% has occurred as the result of employing more capital intensive production processes and 52% has occurred as the result of "Hicks-neutral" technological variegation.

With regard to Objective 3,the CES trend variable specification shows that inter-scalar technological variegation is labour saving (capital augmenting, Solow neutral) and the technological variegation bias is +.072.

With regard to the influence of size and vintage the "Chow test" shows that vintage of production equipment does not have a qualitative influence on the production parameters but size of operation does.

Lastly, with regard to Objective 5, the CES dummy variable specification shows that the elasticity of substitution between capital and labour for all sawmills is .946. This value is insignificantly different from one at the 90% confidence level and therefore the Leontiff hypothesis of zero σ is rejected but the Cobb-Douglas hypothesis of unitary σ cannot be rejected.

B. Evaluation of The Models

A main objective of this thesis is to compare alternative functional forms and in light of the purpose of the analysis, data availability and extent of potential biases,to make recommendations as to the most appropriate functional form for future analysis. The empirical estimation of structurally unique models with the same data base, provides the opportunity of judging the validity of the various

methodologies used in this thesis by comparing results between the models and then evaluating the various results in light of the restrictive assumptions imposed upon the mathematical models used to provide them.

Specifically, the estimation of the relatively more flexible CES functional form provides the opportunity of testing the appropriateness of the Solow model assumptions of: a) linear homogeneity, b) unitary elasticity of substitution and c) Hicks neutral technological differences. The results provided by the CES model suggest that two of the three above assumptions required for the Solow model are not appropriate and therefore there is a high probability of bias in the measures provided by the model. Although the assumption of an elasticity of substitution of one is legitimate, the CES model shows that returns to scale are not constant but declining and that technological variegation is not Hicks neutral but Solow neutral (labour saving). The combined effect of these improprieties is to both underestimate the cross-sectional influence of technological variegation provided by Solow's model, and to improperly categorize it. Because the coefficient (A(s)) cannot be fully isolated, part of the productivity improvement attributed to capital deepening would in fact be capital augmenting technological variegation. Also the fact that returns to scale are declining implies that the Solow relationship is entirely mis-specified because,

Eq. 7.1 $Y = A(s)\ L^{1-\tau} K^{\tau}$

cannot be transformed to

Eq. 7.2 $Y/L = A(s)\ (K/L)^{\tau}$

Thus, although the Solow model does indicate that technological variegation does reflect differences in productivity, the indicated magnitude and nature of the technological variegation is inaccurate. The simplified model therefore cannot be recommended as an appropriate abstraction of the Alberta sawmill industry since the empirical measures provided are misleading in that they do not accurately or even closely reflect production relationships in the industry.

The two specifications of the CES production function also provide the opportunity for comparison. Although both models have been designed to allow for the cross-sectional influence of size on the production relationships, the dummy variable specification is more informative than the trend variable specification.

The dummy variable specification provides a unique set of parameters of production for each predefined size class. Thus quantum differences in the parameters can be ascertained. The trend variable specification on the other hand, provides one set of parameters which are intended to represent the entire industry in aggregate. Significant trend variable coefficients in the factor proportions equation indicate a uniform increase in capital intensity from size class to size class. For the explicit CES equation the trend variable coefficient indicates a uniform rate of disembodied technological variegation.

Observation of the results provided by the two models shows that the trend variable specification results are consistently lower than the results provided by the dummy variable model at all levels of production. Since the data requirements for the two models are virtually identical and since the more flexible dummy variable specification provides more representative results (and a better fit of the model as

indicated by the higher coefficient of determination) this model is superior to the trend variable specification. Thus for analysis intended for deriving the structural production relationships governing Alberta sawmill production, the dummy variable specification is recommended. If on the other hand the purpose of the analysis is to determine the nature and extent of technological variegation, the trend variable specification should be used.

C. Policy Implications of the Results

Through the public decision making process, policies are formulated with the purpose of achieving desirable socio-economic objectives. Desirable objectives may include economic efficiency in production and consumption, an equitable distribution of wealth, income stability, full employment, and/or a healthy rate of growth. These objectives are priorized by society, either implicitly or explicitly, in the political process. Central to decisions regarding the effective implementation of government policy are choices regarding 1) the allocation of publicly owned natural resources, 2) the magnitude and nature of public participation in the industrial complex, 3) the degree of public commitment to research and development.

This thesis provides a more enlightened understanding of the implications of structural production relationships in terms of effective implementation of policy. With regard to the sawmill industry specific instruments of policy which are influenced by these structural relationships include: quota allocation, subsidization, and public investment in research and development. The significance of the structural parameters of technological variegation, returns to scale and

elasticity of substitution are therefore addressed in the succeeding text.

Significance of Technological Change

For the Alberta sawmill industry, general observation of sawmilling methods suggests that technological variegation is partly of a biological nature in increasingly larger sizes of sawmills. This conclusion is made from the observance of a greater preponderance of equipment that is more conducive to enhanced recovery in larger sawmills. However, the empirical evidence provided in this thesis suggests that factor embodied technological differences outweigh "Hicks neutral" technological differences and that cross-sectional technological variegation is actually of a mechanical nature. What then is the significance of this feature in terms of the effectiveness of the previously described instruments of policy? The Solow and the CES models show that technological variegation does contribute to the productivity of inputs in larger mills. The first important point to make is that productivity improvements which occur from the use of newer[47] technologies are the result of research and development efforts which are necessary for the initial invention and refinement of the technologies. Thus in terms of the socio-economic objectives of promoting economic efficiency, generation of wealth and stabilization of incomes, past commitments to R & D have paid rewards in improved productivity and cost competitiveness, which in turn have directly contributed to the achievement of desirable objectives. A continued commitment to R & D and technology transfer will be necessary to

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[47] Larger firms utilize newer technologies whereas smaller firms generally utilize more traditional technologies.

maintain the local industries competitive advantage.

Technology can also be managed through alternative quota allocation schemes. The empirical evidence indicates that technological variegation for larger mills is labour saving. This conclusion comes from the results of the CES trend variable specification which shows a technological variegation bias of +.072. Figure 7.1 illustrates this situation graphically. If stumpage is allocated to mills characterized by isoquant Q (small size technology) then L_1 and K_1 levels of labour and capital are required. If on the other hand the stumpage required to produce 100 MM fbm of lumber is allocated to larger plants (characterized by isoquant Q') then less labour (L_2) and more capital (K_2) is required. In addition the position of the isocost lines indicate that the cost of producing 100 MM fbm with small size technology is higher than the cost with large scale technology (assuming both sizes are faced by the same w/r). Thus, if an objective of policy is to generate employment then allocation of stumpage to small mills is an appropriate measure. However, since allocation of quota to small mills entails a loss in cost efficiency, some form of non-discriminatory subsidy would be necessary to maintain production (and consequently employment) levels. If on the other hand industrial competitiveness and efficiency are more important, allocation of quota to large scale technology is the appropriate course of action. Since this approach improves the industry's competitive advantage, there may be a consequent outward shift in isoquant Q' to a higher level of output. Demand for stumpage would subsequently be increased.

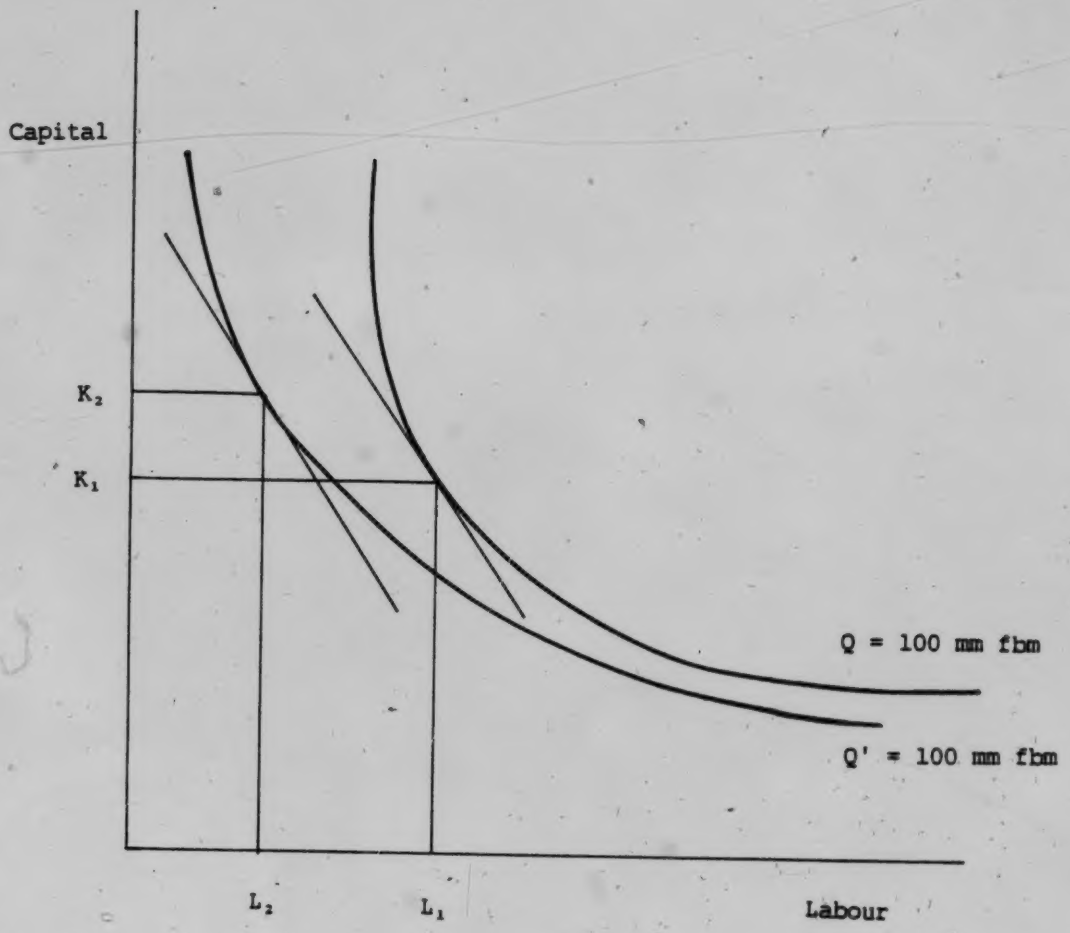

FIGURE 7.1. Effect on Demand for Factors and Cost Effectiveness of Alternative Quota Allocations

Significance of Returns to Scale

Returns to scale are important from a policy standpoint because they are a determinant of optimum plant size. Gray (1981, p. 71) states

> "The optimum scale of sawmilling is determined by two things: the economies of scale in sawmilling itself, which is based on the technology of sawmilling costs and thus is basically independent of local conditions, and the cost of log supply to sawmills of various sizes, which is strongly influenced by local conditions. ... The optimum size of sawmill for any particular area or location must balance economies of scale in sawmilling against the diseconomies of scale of a larger log supply drawn from further afield."

The measures of returns to scale provided in this study are unique for two reasons. First, the industry's production function is not assumed to be homogeneous for all sawmills and therefore production technology describing input-output relationships at different levels of output, is characterized by unique values for returns to scale. Second, the empirical measures of returns to scale provided for each of the four categories of output, implicitly account for the diseconomy influence of larger log supply. This is achieved in the measures of output where wood supply costs have been netted out of the value of total sales. If unit wood supply costs increase proportionately with increasing output then declining returns to scale will be reflected.

An important distinction must be addressed at this point. In the previous section, the implications of biased technological change on inter-scalar quota allocations are pointed out. Quota allocation to large plants is shown to be the most appropriate course of action for

maintaining industrial competitiveness. The significance of the technological change bias in this case is not dependent on the allocation of quota to *existing* large plants. In fact, intuitively the bias might be even more relevant if stumpage is allocated to *new* large plants rather than *new* small plants. With respect to the implications of returns to scale, the significance of the parameters *is* dependent on the provision of additional stumpage to the *existing* industrial infrastructure. If stumpage is allocated to new plants,the significance of differences in returns to scale would not be relevant because the causes of declining returns to scale (which include limited managerial capability, bureaucracy and increased transportation costs) are only influential when expanding existing plants.

The empirical measures of returns to scale indicate that, in terms of efficiency, larger scale plants are preferable to small scale plants for the disposal of excess[**] quota. This feature is illustrated in Figure 7.2 where the increase in labour from L_1 to L_2 and the increase in capital from K_1 to K_2 represent a doubling of both labour and capital. The 100% increase in labour and capital cause a $[(\Delta Q/Q) \times 100]\%$ increase in output. For large mills the empirical evidence indicates that $\Delta Q/Q(\text{large}) = .88$ and for the smallest firms $\Delta Q/Q(\text{small}) = .65$ when both inputs are doubled. Thus the pervasive policy tradeoff between efficiency and employment generating potential is once again illustrated. The magnitude of returns to scale is shown to have significance with regard to the implication of alternative allocations of quota. For example, the allocation of a particular level of stumpage will create more jobs if provided to existing small plants than if

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[**] Excess quota is the portion of the AAC which is presently unallocated.

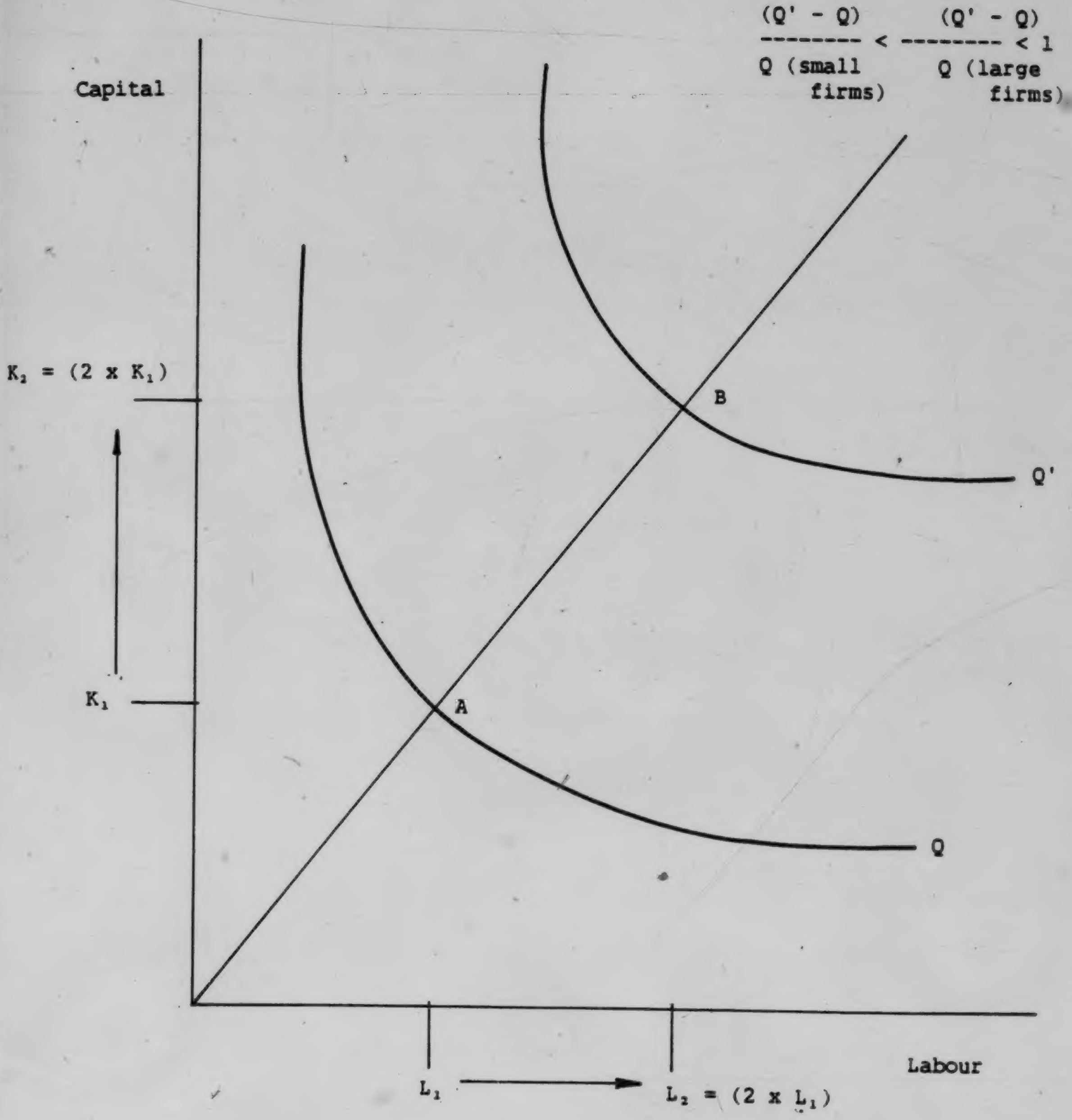

FIGURE 7.2. Implications of Returns to Scale on Alternative Quota Allocations

allocated to existing large plants. However, the larger plants will be more able to convert the stumpage in a cost efficient manner.

Thus four separate directions with respect to alternative quota allocation possibilities are identified. The significance of technological change and returns to scale will depend on whether quota is allocated to existing large plants, new large plants, existing small plants or new small plants.

Significance of the Elasticity of Factor Substitution

The final parameter which has relevance in terms of the effectiveness of policy instruments is the elasticity of substitution (σ). The precise empirical measure of this parameter is .946. This value is insignificantly different from one at the 90% confidence level. The measure is invariant to scale and is thus homogeneous for all Alberta sawmills.

As described in Chapter III σ is effectively a measure of the ease with which factors of production can be substituted for one another. The parameter is a measure of the shape of the firm's isoquant and is thus of practical significance for analysis, the purpose of which is to evaluate the effect of changes in the price of a factor of production. An increase in the price of a particular factor input will have two effects on the overall quantity of the factor demanded: an output effect and a substitution effect (Layard and Walters, 1978). The magnitude of the substitution effect is fully dependent on the direct elasticity of substituion between the two factors of production.

The pragmatic significance of the magnitude of σ to individuals concerned with economic policy is brought to bear in decisions regarding

the effective implementation of subsidization programs (particularly when the program is oriented toward subsidizing a particular input). For example, when a labour subsidization program (designed to maintain or generate employment) is implemented, the effect is to reduce labour's wage relative to the capital wage. This influence reduces the slope of the isocost line causing labour to be substituted for capital along the isoquant. The effectiveness of a given amount of subsidy in causing the substitution to occur, is entirely dependent on the slope of the isoquant (defined by the elasticity of substitution). Figure 7.3 illustrates the significance of σ in terms of the effective use of subsidy programs. The optimum level of labour for both isoquants QQ and Q'Q' at the initial factor price ratio (w_1/r_1) is at L_1. Since a subsidization of labour reduces the cost of a unit of labour the slope of the isocost line becomes flatter to w_2/r_1. The shapes of isoquants QQ and Q'Q' indicate that σ for the former is higher than σ for the latter. Since substitution is easier along QQ a given labour subsidy creates more labour (L_2) than would be created if the industry isoquant were represented by Q'Q' (L_3). Moreover, since production along isoquant Q'Q' entails a higher cost, the output effect generated by the subsidy would necessarily be lower.

In relation to measures of the elasticity of substitution provided for the U.S. lumber and wood products industry (e.g., Steir, 1980) the value of σ for the Alberta sawmill industry is higher, indicating that input substitution is easier in the local industry. For comparison, a 10% change in the factor price ratio will cause roughly a 10% change in factor proportions in the Alberta sawmill industry whereas a 10% change

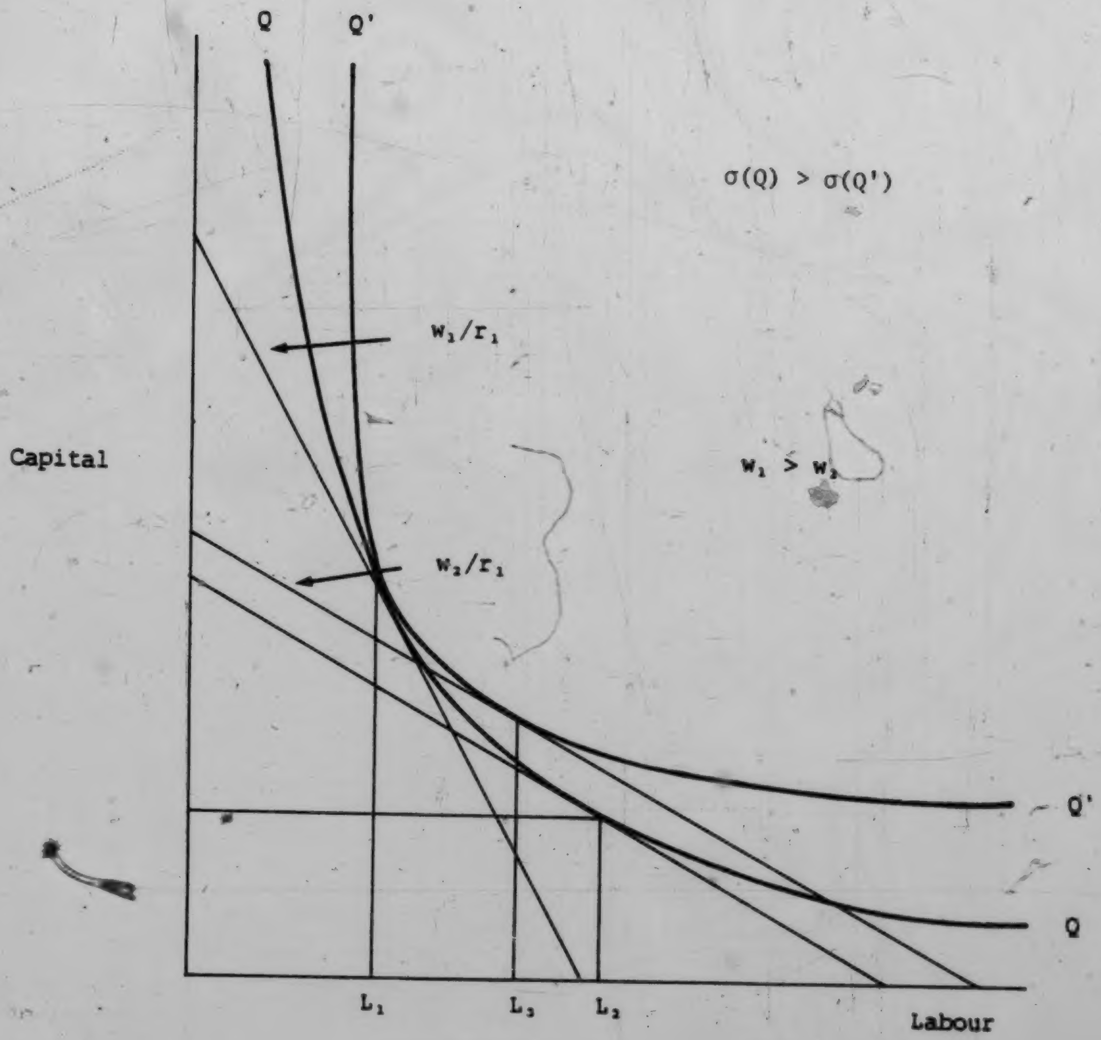

FIGURE 7.3. Implication of Elasticity of Substitution on Effectiveness of Labour Subsidization

in the U.S. lumber and wood products industry causes a roughly 6.5%[••] change in factor proportions.

D. Areas for Further Research

The progressively sophisticated techniques employed in this thesis have roughly paralleled the chronological development of neoclassical production economics. First, the basic Cobb-Douglas production function was estimated. Second, Solow's model for evaluating the influence of technological variegation was applied. Finally, various specifications of the constant elasticity of substitution (CES) production function were rendered. The latter functional form is more flexible than previous forms because a priori restrictions are not placed on the value of σ. The CES form however is still confined to consideration of production with two factor inputs which must be substitutes. Layard and Walters (1978, p.265) describe this case with

> "However, if there are only two factors, each must be a substitute for the other (output constant).The two-factor case thus misses many of the important issues that arise - does a rise in the supply price of unskilled labour raise or lower the demand for skilled labour (it could do either)? We therefore move next to the general (three or more factor) case and develop the Allen elasticity of substitution, which, though less intuitively simple is in the general case more practically useful than the 'direct' elasticity."

The transcendental logarithmic production function attributed to Christensen, et al. (1971) provides the flexibility required for

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[••] This value is from Sato (1977).

evaluating production with greater than two inputs. Further analysis with this more flexible functional form is a natural extension of the research endeavors in this dissertation. The developed model representing Alberta sawmill production could conceivably be expanded to include not only the traditional inputs of capital and labour but also stumpage and energy. Thus the nature of substitution (or complementary) relationships between all four inputs can be identified.

An additional direction requiring investigation is to empirically separate the productivity improving influences of managerial capability from the factor or product augmenting influences of diverse technologies. In other words, what is required is to quantify cross-sectional productivity improvements and to determine the relative contributions of technical efficiency and technological variegation. A possible initial approach to this would be to incorporate dummy variables into a simple Cobb-Douglas production function to reflect the qualitative influences of both more efficient technology and enhanced managerial capability. Dummy variables designed to reflect the effect of diverse technologies could be sensitive to vintage of production equipment. Dummy variables for reflecting managerial capability could be sensitive to length of ownership. A study of this nature would do much to rationally explain the reasons why a large number of firms, with diverse technologies, can co-exist in a perfectly competitive market environment.

E. Conclusion

This thesis has hopefully provided a more enlightened appreciation of the structural characteristics of the Alberta sawmill industry. These structural characteristics are defined by a series of parameters which describe the relationships between inputs and output and the inter-relationships between inputs themselves. As has been illustrated, the nature of these relationships and inter-relationships have important implications with regard to the effectiveness of certain instruments of policy. A primary emphasis of the thesis has been to illuminate the cross-sectional diversity of the industry.

The assumption that the information provided in this study is unquestionably conclusive would perhaps be risky. The form of research embarked on in this study is the first of its kind applied to the Alberta sawmill industry. The empirical findings therefore require substantiation. However, in general terms, the findings are useful as broad descriptive or diagnostic benchmarks and their use as such is likely valid.

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Appendix A. Proof of Linear Homogeneity Conditions

Proof of Y/L = f (K/L) Relationship

The function $y = f(K,L)$ is homogeneous of degree 1 or displays constant returns to scale if:

$$f(\lambda K, \lambda L) = \lambda Y$$

Setting $\lambda = 1/L$ transforms the above relation to;

$$\frac{Y}{L} = f \; \frac{K}{L},1$$

Thus with constant returns to scale output per unit labour depends solely on capital per unit labour.

Appendix B. Proof of Cost Minimization Conditions

Mathematical Proof of Necessary and Sufficient Conditions For Profit Maximization[50]

Assuming firms are cost minimizers the objective function is to minimize;

$TC = f(w, L, r, K)$

Subject to the technical constraint imposed by the production function;

$y = f(K,L)$

The Lagrangian equation appears as;

$L = TC + \lambda \{ y - f(K,L) \}$

$L = (w \cdot L) + (r \cdot K) - \lambda \{y - f(K,L)\}$

Cost minimization is insured by differentiating with respect to K, L, and λ and setting the resulting relations equal to zero. Thus the necessary conditions for an extreme value are represented by:

a) $\partial L/\partial l = w - \lambda MPl = 0$

b) $\partial L/\partial k = r - \lambda MPk = 0$

b) $\partial L/\partial \lambda = f(K,L) - y = 0$

A constant level of output is ensured by setting the last equation to zero. Therefore, any changes in capital and labour result in movements along the isoquant which is described by the constraints imposed by the production function ($y = f(K,L)$). The relationship between the MRTS and the factor price ratio is shown by taking the ratio of a) and b);

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[50] See: Doll, J.P. and F. Orozem. 1978. Production Economics: Theory with Applications, Grid Inc.

$$\frac{w}{r} = \frac{\lambda\ MPl}{\lambda\ MPk} = MRTS$$

Therefore, when the ratio of the marginal products equals the factor price ration, then the costs of producing a particular output are minimized.

Appendix C. Detailed Statistical Tests

TABLE C.1

Statistics Required for Testing the Homogeneity of the Sample and Population Means and Variances of Three Variables

	Production		Average Haul		Employment	
	Sample	Pop.	Sample	Pop.	Sample	Pop.
ΣX, ΣY	368152	789368	2399	4320	16,312	33,219
n	83	183	83	183	83	183
$\bar{x}$, $\bar{y}$	4435	4313	28.9	23.6	196	181
$\Sigma(x-\bar{x})^2$, $\Sigma(y-\bar{y})^2$	10,181,378,000	31,385,780,000	62,444	133,401	14,770,460	39,973,289
S^2x, S^2y	124,163,146	172,449,341	761	733	180,128	219,633

The formula for deriving the F statistic is equivalent to;

$$F^* = \frac{Sy^2}{Sx^2}$$

where $Sx^2 = \frac{\Sigma(x-\bar{x})^2}{n_1-1}$ and $Sy^2 = \frac{\Sigma(y-\bar{y})^2}{n_2-1}$

The formula for deriving the t statistic is equivalent to;

$$t^* = \frac{(\bar{x} - \bar{y})}{S\bar{x}-\bar{y}}$$

where

$$S(\bar{x}-\bar{y}) = \sqrt{S^2x-S^2y}$$

$$S\bar{x} = S / \sqrt{n_1} \qquad , Sy = S / \sqrt{n_2}$$

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1 + n_2 - 2}}$$

For the production variable the calculated F statistic is;

$F^* = 172{,}449{,}341 / 124{,}163{,}146 = 1.39$

$F_{95}\,(_{182\;82}) = 1.39$

Since the calculated F statistic for the production variable does not exceed the critical value of F the assumption of homogeneous variance is acceptable.

For the average haul variable the F statistic is;

$F^* = 761 / 733 = 1.038$

$F_{95}\,(_{182\;82}) = 1.39$

Thus the sample and population variances are homogeneous.

For the employment variable the F statistics is:

$F^* = 219{,}\ 633 / 180{,}128 = 1.22$

$F_{95}\,(_{182\;82}) = 1.39$

Thus the sample and population variances are homogeneous. In fact, the variances of all three variables are homogeneous.

For the production variable the value of t* is;

$$S = \sqrt{\frac{124{,}163{,}146 + 172{,}449{,}341}{83 + 183 - 2}} = \sqrt{157{,}451{,}356}$$

$$S_{\bar{x}} = \sqrt{\frac{157{,}451{,}356}{\sqrt{183}}} = \sqrt{1{,}897{,}004}$$

$$S_{\bar{y}} = \sqrt{\frac{157{,}451{,}356}{\sqrt{83}}} = \sqrt{860390}$$

$$S_{\bar{x}-\bar{y}} = \sqrt{1{,}897{,}004 + 860{,}390} = 1660$$

$$t^* = \frac{4.436 - 4.313}{1660} = .074$$

$$t_{,5}\ ({}_{264}) = 1.97$$

Since $t^* < t(264)$ the means are equivalent.

For the average haul variable the value of t* is;

$$S = \sqrt{742}$$

$$S_{\bar{x}} = \sqrt{8.9}$$

$$S_{\bar{y}} = \sqrt{4.0}$$

$$S_{\bar{x}-\bar{y}} = \sqrt{12.9} = 3.59$$

$$t^* = \frac{28.9 - 23.6}{3.59} = 1.48$$

Since $t^* < t(264)$ the means are equivalent.

For the employment variable the value of t* is;

$$S = \sqrt{207363}$$

$$S_{\bar{x}} = \sqrt{2498}$$

$$S_{\bar{y}} = \sqrt{1133}$$

$$S_{\bar{x}-\bar{y}} = \sqrt{3631} = 60.27$$

$$t^* = \frac{196 - 181}{60.27} = .25$$

Since $t^* < t({}_{264})$ the two means are again equivalent.